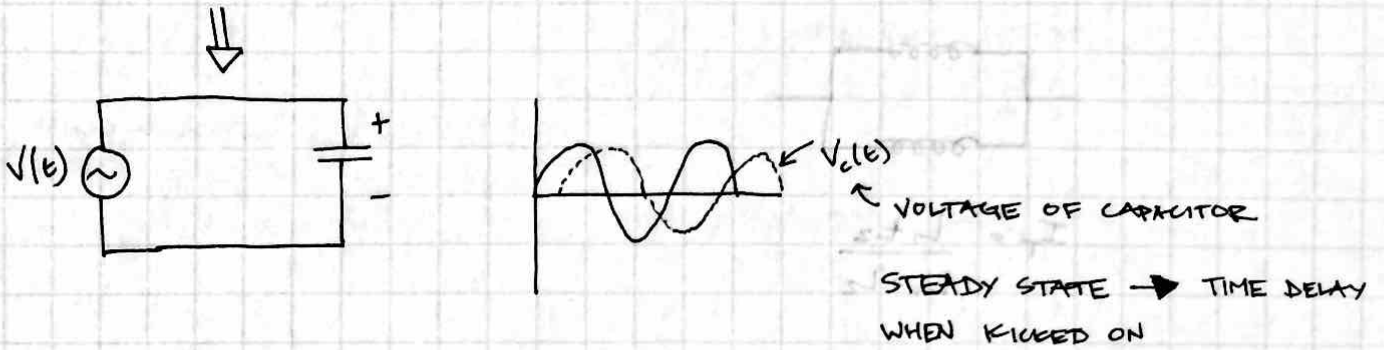
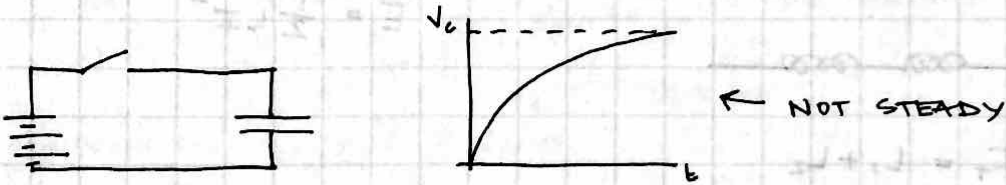


JANUARY 10, 2012

ENGR 202 STEADY STATE SINUSOIDAL RESPONSE OF LINEAR CIRCUITS



GIVEN A FREQUENCY, WHATS THE PHASE AND VOLTAGE?

RESISTORS → CAPACITORS → INDUCTORS

RESISTOR: [OHM] Ω

OHMS LAW: $V = IR$

PARALLEL $R_T = \frac{R_1 R_2}{R_1 + R_2}$

WATTS LAW: $P = VI = I^2 R = \frac{V^2}{R}$

CAPACITOR: 'OR'

UNIT: [FARAD] F

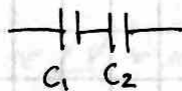
← POLAR CAPACITOR, SAME FOR ANALYSIS PURPOSES.

$Q = CV$
 ↑ CAPACITANCE ↑ VOLTAGE

$\frac{dQ}{dt} = C \frac{dV}{dt}$

$E = \frac{1}{2} CV^2$

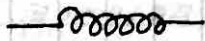
↑ $I = \frac{dQ}{dt}$



$C_{TOTAL} = \frac{C_1 C_2}{C_1 + C_2}$

PARALLEL = $C_1 + C_2$

INDUCTORS:



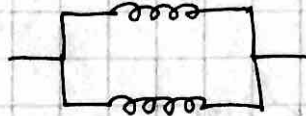
[HENRY]

$$V = L \frac{dI}{dt}$$

$$E = \frac{1}{2} LI^2$$



$$I_T = L_1 + L_2$$

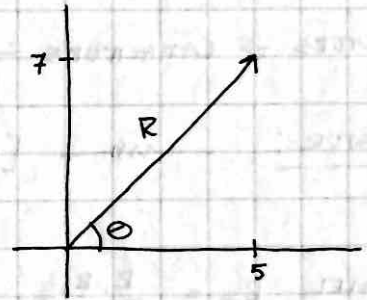


$$I_T = \frac{L_1 L_2}{L_1 + L_2}$$

COMPLEX NUMBERS:

$$j = \sqrt{-1} \quad j^2 = -1 \quad j^3 = -j$$

$$Z = X + jy$$



$$Z = 5 + j7 \quad \text{or}$$

$$Z = R \angle \theta$$

READING NOTES (CHAPTER 9)

A SINUSOIDAL VOLTAGE :

$$v(t) = V_E \sin(\omega t)$$

$$T = \frac{2\pi}{\omega} \text{ IS THE PERIOD}$$

$$V_E = \text{AMPLITUDE}$$

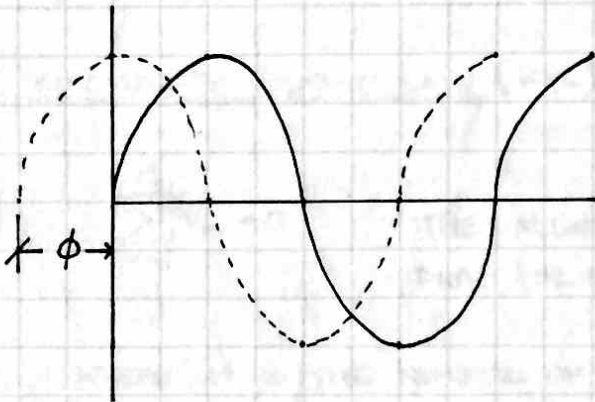
A FUNCTION IS PERIODIC IF:

$$v(t + T) = v(t)$$

$$f = \frac{1}{T} \text{ IS THE FREQUENCY}$$

MORE GENERAL VERSION :

$$v(t) = V_m \sin(\omega t + \phi) \text{ WHERE } \phi \text{ IS THE PHASE}$$



ADDITION OF SINUSOIDS :

$$A \cos \omega t + B \sin \omega t = C \cos(\omega t - \theta) \text{ WHERE } C = \sqrt{A^2 + B^2} \text{ AND}$$

$$\theta = \tan^{-1}\left(\frac{B}{A}\right)$$

PHASORS : A PHASOR IS A COMPLEX NUMBER THAT REPRESENTS THE PHASE AND AMPLITUDE OF A SINUSOID

$$Z = x + jy \text{ WHERE } j = \sqrt{-1}$$

$$Z = r \angle \phi = r e^{j\phi} \text{ WHERE } r \text{ IS THE MAGNITUDE OF } Z \text{ AND } \phi \text{ THE PHASE}$$

ENGR 202 REVIEW NOTES (READING):

KIRCHHOFF'S LAWS:

KIRCHHOFF'S CURRENT LAW (KCL):

$$\sum_{n=1}^N i_n = 0$$

THE ALGEBRAIC SUM OF CURRENTS ENTERING A NODE (OR CLOSED BOUNDARY) IS ZERO

WHERE N IS THE NUMBER OF BRANCHES CONNECTED TO THE NODE

"THE SUM OF CURRENTS ENTERING THE NODE IS EQUAL TO THE SUM OF CURRENTS LEAVING THE NODE"

KIRCHHOFF'S VOLTAGE LAW (KVL)

$$\sum_{m=1}^M v_m = 0$$

THE ALGEBRAIC SUM OF ALL VOLTAGES AROUND A CLOSED PATH (OR LOOP) IS ZERO

WHERE M IS THE NUMBER OF VOLTAGES IN THE LOOP (OR # OF BRANCHES)

"SUM OF VOLTAGE DROPS = SUM OF VOLTAGE RISES"

SERIES RESISTORS:

$$R_{EQ} = R_1 + R_2 + \sum_{n=1}^N R_n \quad \text{WHERE} \quad v_1 = \frac{R_1}{R_1 + R_2} V \quad \text{AND} \quad v_2 = \frac{R_2}{R_1 + R_2} V$$

IN GENERAL, FOR N RESISTORS IN SERIES, WITH SOURCE VOLTAGE V ,

$$v_n = \frac{R_n}{R_1 + R_2 + \dots + R_n} V \quad \text{THIS IS THE VOLTAGE DIVIDER EQUATION}$$

ENGR 202 READING REVIEW CONT.

PARALLEL RESISTORS:

$$R_{EQ} = \frac{R_1 R_2}{R_1 + R_2} \quad \text{FOR TWO RESISTORS IN PARALLEL}$$

$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N} \quad \text{FOR MULTIPLE RESISTORS IN PARALLEL}$$

$$i_1 = \frac{R_2}{R_1 + R_2} i ; \quad i_2 = \frac{R_1}{R_1 + R_2} i \quad \text{CURRENT DIVIDER EQUATION}$$

METHODS OF ANALYSIS:

* **NODAL ANALYSIS:** BASED ON SYSTEMATIC APPLICATION OF KIRCHOFFS CURRENT LAW. INTERESTED IN FINDING THE NODE VOLTAGES

STEPS TO DETERMINE NODE VOLTAGES:

- 1.) SELECT NODE AS REFERENCE, ASSIGN VOLTAGE VARIABLES TO THE REMAINING NODES (N-1)
- 2.) APPLY KCL TO EACH OF THE N-1 NON-REFERENCE NODES, USE OHMS LAW TO EXPRESS BRANCH CURRENTS IN TERMS OF NODE VOLTAGES
- 3.) SOLVE THE RESULTING SIMULTANEOUS EQUATIONS

CURRENT FLOWS FROM A HIGHER POTENTIAL TO A LOWER POTENTIAL

$$i = \frac{V_{HIGHER} - V_{LOWER}}{R}$$

PASSIVE SIGN CONVENTION: IS SATISFIED WHEN CURRENT ENTERS THROUGH THE POSITIVE TERMINAL (SUPPLYING) AND EXITS THROUGH POSITIVE ABSORBING.

NODAL ANALYSIS CONT.

NODAL ANALYSIS W/ VOLTAGE SOURCES

CASE 1: IF THE VOLTAGE SOURCE IS CONNECTED BETWEEN THE REFERENCE NODE AND NON-REFERENCE NODE, VOLTAGE OF NON-REFERENCE NODE = VOLTAGE OF VOLTAGE SOURCE.

CASE 2: IF THE VOLTAGE SOURCE (DEPENDENT OR INDEPENDENT) IS CONNECTED BETWEEN TWO NON-REFERENCE NODES, THE TWO N-R NODES FORM A SUPERNODE.

SUPERPOSITION: BASED ON LINEARITY PRINCIPLE, DETERMINES CONTRIBUTION OF EACH INDEPENDENT SOURCE, THEN ADDS THEM TO DETERMINE FINAL TOTAL.

STEPS TO APPLY SUPERPOSITION:

- 1.) "TURN OFF" ALL INDEPENDENT SOURCES EXCEPT ONE. FIND V OR I USING NORMAL METHODS.
- 2.) REPEAT STEP 1 FOR EACH OTHER INDEPENDENT SOURCE
- 3.) FIND ALL OTHER COMBINATIONS BY ADDING ALGEBRAICALLY ALL CONTRIBUTIONS.

VOLTAGE SOURCE "OFF" = SHORT CIRCUIT

CURRENT SOURCE "OFF" = OPEN CIRCUIT.

JANUARY 17, 2012

GENERALLY SHOULD KEEP PHASOR ANGLES BETWEEN $\pm 180^\circ$ AND 180°
- ALWAYS CHECK GEOMETRY FOR \tan^{-1} SOLUTIONS

RESPONSE TO SINUSOIDAL SIGNALS

RESISTORS:

$$V = iR \text{ OR } i = \frac{V}{R} = \frac{A}{R} \sin(\omega t)$$

CAPACITOR:

$$I = C \frac{dV}{dt} = C A \cos(\omega t) \omega = \omega C A \sin(\omega t + 90^\circ)$$

INDUCTOR:

$$V = L \frac{dI}{dt} \Rightarrow I = \frac{1}{L} \int V dt \Rightarrow I = \frac{-A}{\omega L} \cos(\omega t) = \frac{A}{\omega L} \sin(\omega t - 90^\circ)$$

PHASOR NOTATION:

$$V(t) = V_p \cos(\omega t + \phi)$$

NOTE: COS CONSIDERED 'BASE' FUNCTION, DUE TO REAL PART IN EULERS EQUATION

EULER

↓

$$V_p \operatorname{Re}\{e^{j(\omega t + \phi)}\} \Rightarrow \operatorname{Re}\{V_p e^{j\omega t} e^{j\phi}\}$$

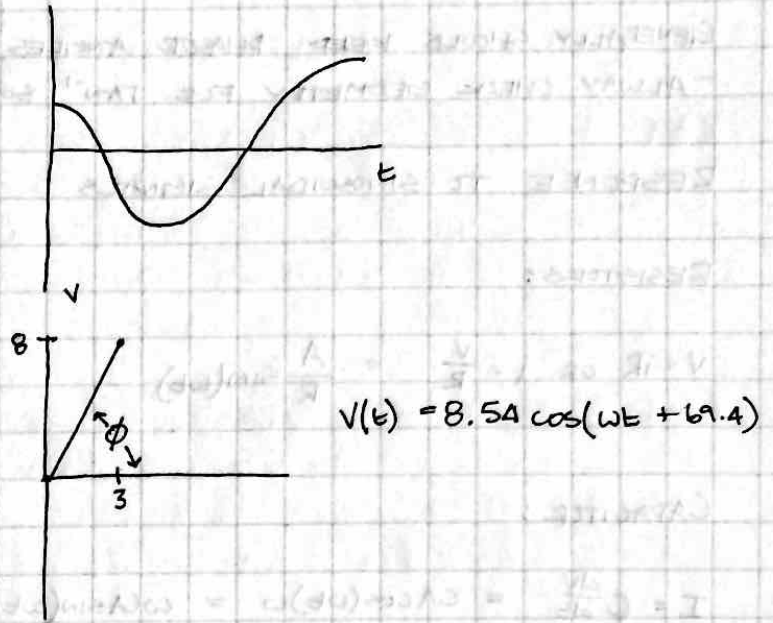
ONLY THING THAT CHANGES = COMPLEX NUMBER

→ DOES CHANGE

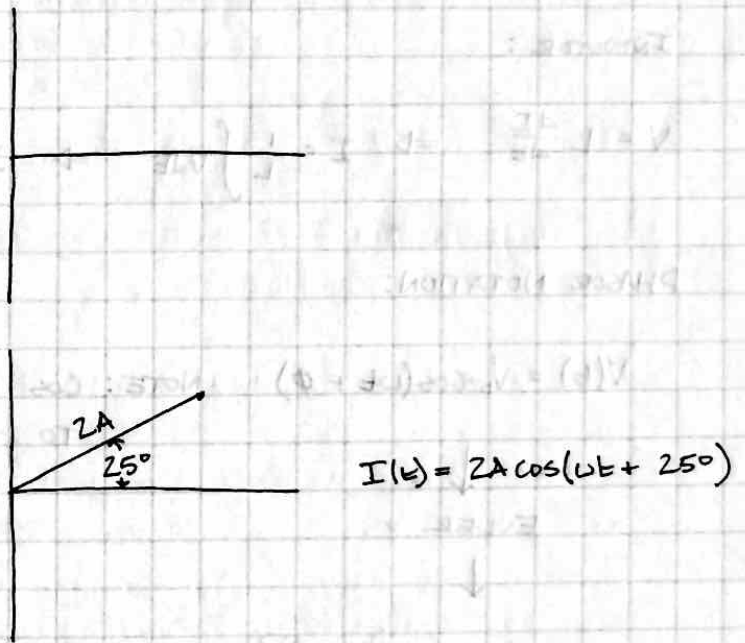
$$\bar{V} = V_p e^{j\phi}$$

PHASORS CONTINUED:

$$\bar{V} = (3 + j8) \text{ VOLTS}$$



$$\bar{I} = 2A \angle 25^\circ$$



$$\bar{V}_2 = 3V e^{j30^\circ} \Rightarrow V(t) = 3V \cos(\omega t + 30^\circ) \Rightarrow \bar{I}_2 = j 50 \text{ AMP}$$

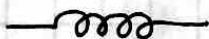
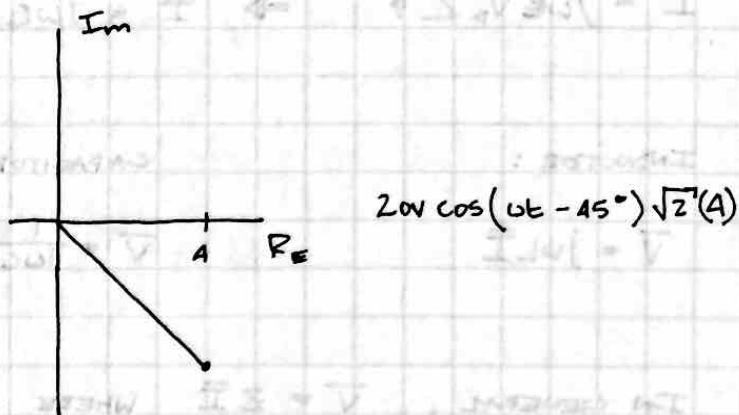
$$\bar{V}_3 = 5V \Rightarrow V_3(t) = 5V \cos(\omega t)$$

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VALUES MUST BE IN RADIAN FOR EULERS IDENTITY

JANUARY 19, 2012

$$\bar{V} = 20V (4 - 4j) \Rightarrow$$



INDUCTOR:

$$V = L \frac{di}{dt}$$

$$i(t) = I_p \cos(\omega t + \phi)$$

$$= \text{Re}(I_p e^{j\omega t} e^{j\phi}) \Rightarrow \bar{I} = I_p \angle \phi$$

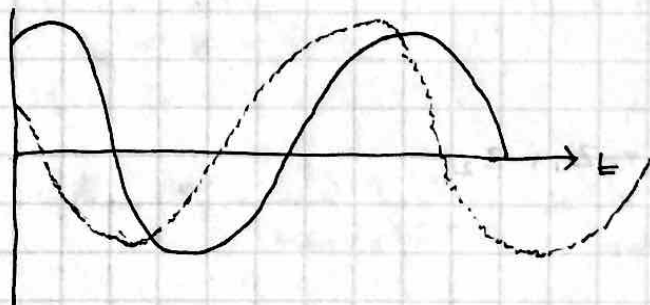
$$v(t) = -\omega L \sin(\omega t + \phi) = -\omega L I_p \sin(\omega t + \phi)$$

$$= \text{Re}(I_p \omega L e^{j\omega t} e^{j\phi} e^{j90^\circ})$$

$$= \text{Re}(I_p \omega L e^{j\omega t} e^{j\phi} j)$$

$$\bar{V} = j\omega L I_p \angle \phi$$

$$\boxed{\bar{V} = j\omega L \bar{I}}$$



"ELI THE ICE MAN"

CAPACITOR:

$$i = C \frac{dv}{dt} \quad \bar{V} = V_p \angle \phi$$

$$\bar{I} = j\omega C V_p \angle \phi \Rightarrow \bar{I} = j\omega C \bar{V}$$

INDUCTOR:

$$\bar{V} = j\omega L \bar{I}$$

CAPACITOR:

$$\bar{V} = \frac{1}{j\omega C} \bar{I}$$

RESISTOR:

$$\bar{V} = R \bar{I}$$

IN GENERAL, $\bar{V} = Z \bar{I}$ WHERE Z IS IMPEDENCE

$$Z = R + jX$$

RESISTANCE

REACTANCE

Z = COMPLEX (UNITS, OHMS Ω)

R = REAL

X = REAL (UNITS, OHMS Ω)

POSITIVE REACTANCE = INDU

$\frac{1}{Z} =$ ADMITTANCE (S, SIEMENS)
OR Y

$$Y = G + jB$$

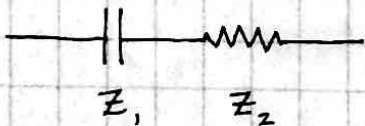
WHERE G - CONDUCTANCE

B - SUSCEPTANCES

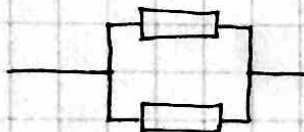
$$Z_{\text{INDUCTOR}} = j\omega L$$

$$Z_{\text{CAPACITOR}} = \frac{1}{j\omega C}$$

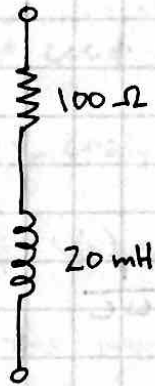
"THE IMPEDENCE TELLS YOU HOW HARD IT IS TO PUSH CURRENT THROUGH SOMETHING"



$$Z_{\text{TOTAL}} = Z_1 + Z_2$$



$$Z_{\text{TOTAL}} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$



FREQUENCY MUST BE GIVEN.

DETERMINE Z_{TOT}

$$f = 400 \text{ Hz}$$

SOLUTION:

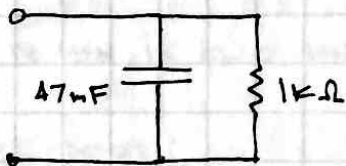
$$Z_{TOT} = Z_1 + Z_2$$

$$Z_1 = R = 100 \Omega$$

$$Z_2 = j\omega L = j2\pi 400 \text{ Hz } 20 \text{ mH} = 50 \Omega$$

WHERE $\Omega = 2\pi f$

$$Z_{TOT} = 100 + j50 \Omega$$



$$f = 1 \text{ kHz}$$

$$Z_{TOT} = ?$$

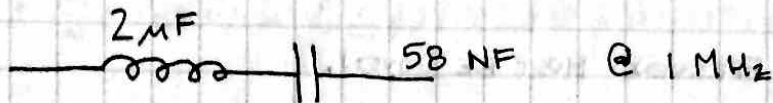
$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Z_1(\text{cap}) = \frac{1}{j\omega C} = -j3.39 \Omega$$

NEGATIVE INDICATED CAPACITOR

$$Z_2 = R = 1 \text{ k}\Omega$$

$$Z_{TOT} = \frac{-j3.39 \Omega \cdot 1 \text{ k}\Omega}{1 \text{ k}\Omega - j3.339 \Omega} = 0.011 - 3.387j \Omega$$



$$Z_{TOT} = ?$$

$$Z_{TOT} = j\omega L + \frac{1}{j\omega C} \quad \text{OR} \quad Z_{TOT} = j\omega L - j\frac{1}{\omega C}$$

(ALL COMPLEX — NO RESISTOR)

$$Z_{TOT} = j2\pi \cdot 10^6 \frac{1}{\text{SEC}} 2\text{MH} - j \frac{1}{2\pi \cdot 10^6 \frac{1}{\text{SEC}} \cdot 58\text{nF}}$$

$$Z_{TOT} = j9.82 \Omega$$

CHAPTER 9 READING NOTES:

* $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$ WHERE $\operatorname{Re}(e^{j\phi}) = \cos\phi$ AND $\operatorname{Im}(e^{j\phi}) = \sin\phi$

$$v(t) = V_m \cos(\omega t + \phi) = \operatorname{Re}(V_m e^{j(\omega t + \phi)}) = \operatorname{Re}(V_m e^{j\phi} e^{j\omega t})$$

THUS; $v(t) = \operatorname{Re}(\bar{V} e^{j\omega t})$ WHERE $\bar{V} = V_m e^{j\phi} = V_m \angle \phi$

* \bar{V} IS THE PHASOR REPRESENTATION OF $v(t)$. A PHASOR IS A COMPLEX REPRESENTATION OF THE MAGNITUDE AND PHASE OF A SINUSOID.

* SWITCHING BACK AND FORTH: PHASORS AND SINUSOIDS

TO OBTAIN THE SINUSOID FROM THE PHASOR, MULTIPLY THE PHASOR BY THE TIME FACTOR $e^{j\omega t}$ AND TAKE THE REAL PART.

TIME-DOMAIN REPRESENTATION:

PHASOR DOMAIN REPRESENTATION:

$$v(t) = V_m \cos(\omega t + \phi) \iff \bar{V} = V_m \angle \phi$$

PHASOR DOMAIN IS ALSO KNOWN AS THE FREQUENCY DOMAIN

TIME-DOMAIN:

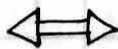
PHASOR DOMAIN:

$$\frac{dv}{dt}$$



$$j\omega \bar{V}$$

$$\int v dt$$



$$\frac{\bar{V}}{j\omega}$$

* CHARACTERISTICS OF $v(t)$ AND \bar{V}

- 1.) $v(t)$ IS INSTANTANEOUS OR TIME-DOMAIN, \bar{V} IS FREQUENCY OR PHASOR-DOMAIN
- 2.) $v(t)$ IS TIME DEPENDENT, WHILE \bar{V} IS NOT
- 3.) $v(t)$ IS ALWAYS REAL W/ NO COMPLEX PART. \bar{V} IS GENERALLY COMPLEX

READING NOTES CONT.

* PHASOR RELATIONSHIPS FOR CIRCUIT ELEMENTS:

- RESISTOR:

$$\bar{V} = IR$$

- INDUCTOR:

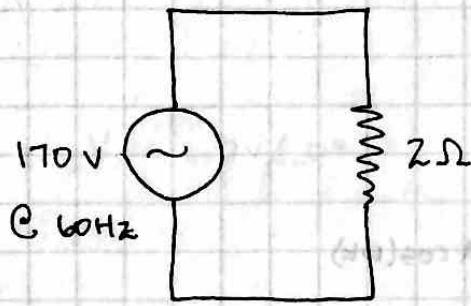
$$\bar{V} = j\omega L I$$

- CAPACITOR:

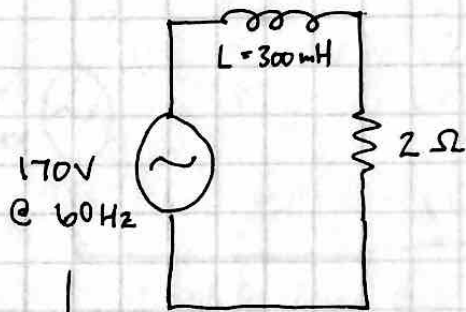
$$\bar{V} = \frac{I}{j\omega C}$$

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EXAMPLE



85A @ 60 Hz



$$L_0 \rightarrow 170V \cos(2\pi 60 \frac{1}{\text{sec}} t) = V(t)$$

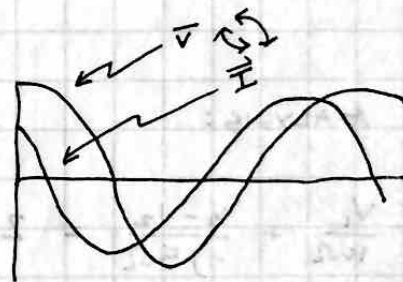
$$I = \frac{V}{Z}$$

$$Z_{RES} = 2 \Omega$$

$$Z_{IND} = j\omega L = j2\pi 60 (0.3)$$

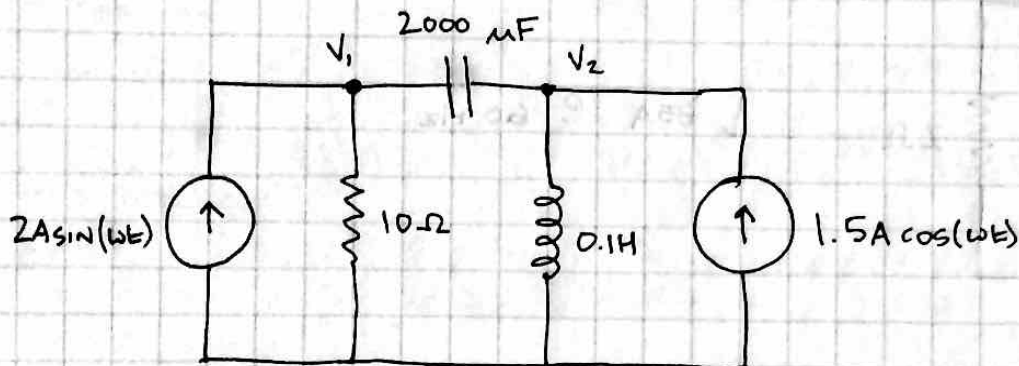
$$Z_{TOT} = (2 + j113.1) \Omega = 113 \angle 89^\circ$$

$$I = 1.5 \angle -89^\circ \text{ A}$$



INDUCTORS LIMIT CURRENT WITHOUT WASTING POWER

EXAMPLE



LOOKING FOR VOLTAGE @ V_1, V_2

$$2A \angle -90$$

$$1.5 \angle 0$$

$$Z_{CAP} = \frac{1}{j\omega C} = -j5 \Omega$$

$$Z_{IND} = j\omega L = j10 \Omega$$

NODE ANALYSIS:

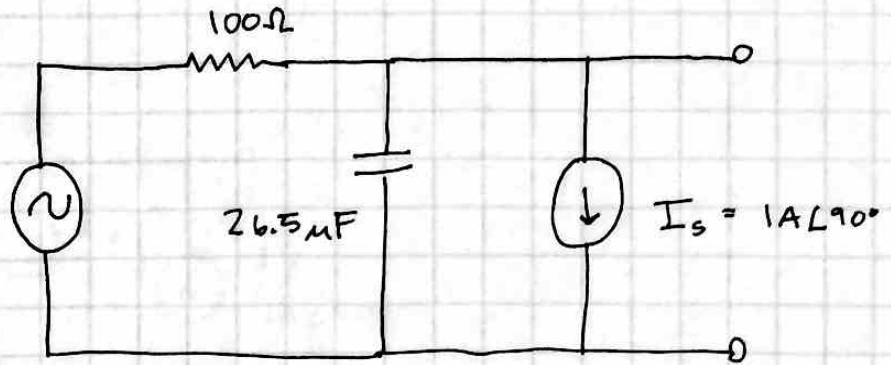
$$1.) \quad \frac{V_1}{10 \Omega} + \frac{V_1 - V_2}{-j5 \Omega} = 2A \angle -90^\circ$$

$$2.) \quad \frac{V_2 - V_1}{-j5 \Omega} + \frac{V_2}{j10 \Omega} = 1.5 \angle 0$$

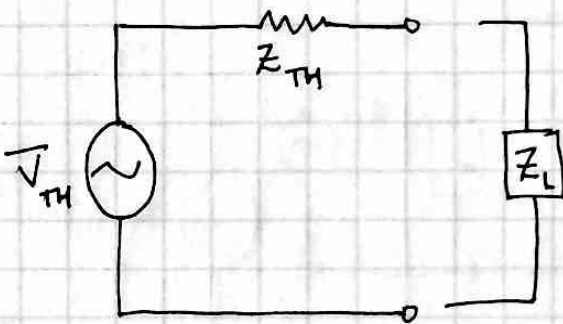
$$\Rightarrow \text{SOLVE FOR } V_1 / V_2 \quad \Rightarrow \quad V_1 = 14 + j8 = 16.1 \angle 29.7^\circ$$

EXAMPLE

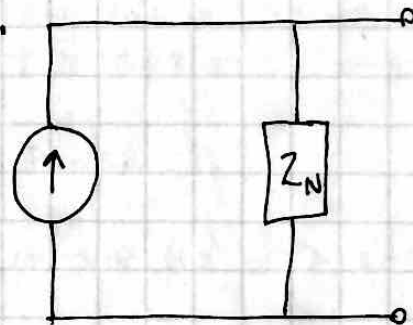
$V_s = 100V \angle 0^\circ$



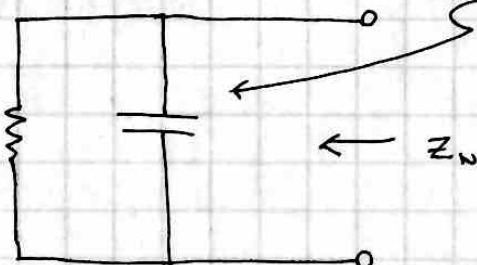
THEVENIN :



NORTON :



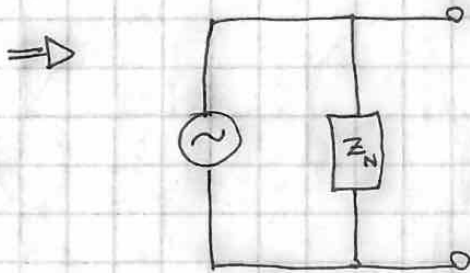
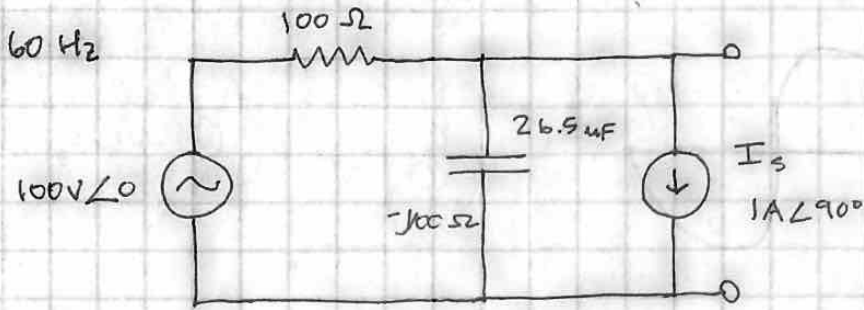
FIND Z_{TH} / Z_N



$$\frac{1}{j\omega C} = -j100\Omega$$

$$Z_N = \frac{(100\Omega)(-j100\Omega)}{(100\Omega) - (j100\Omega)} = 50 - j50\Omega$$

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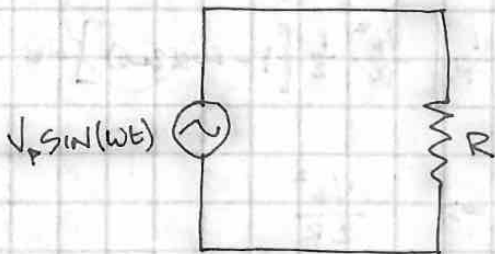


$$V_{TH} = \bar{I}_N Z_N$$

NEW HW ON BLACKBOARD
- 6 PROBLEMS ("MORE INVOLVED")

* POWER

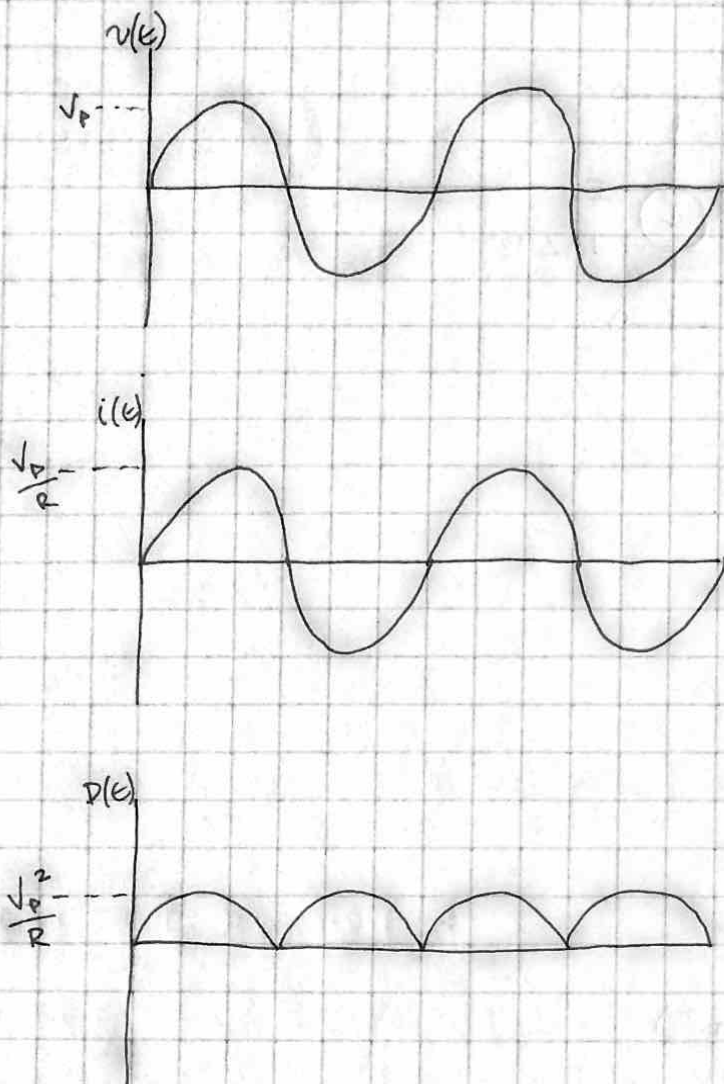
$$P = v(t)i(t) \Rightarrow \text{SO} \Rightarrow P(t) = v(t)i(t)$$



$$v(t) = V_p \sin(\omega t)$$

$$i(t) = \frac{V_p}{R} \sin(\omega t)$$

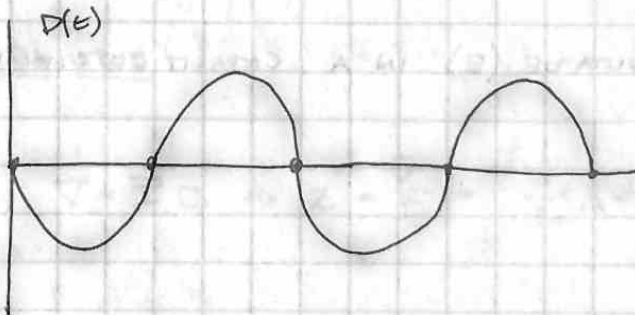
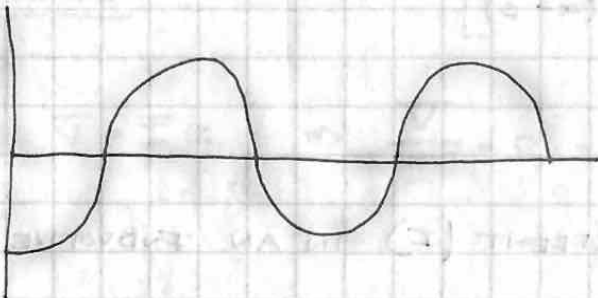
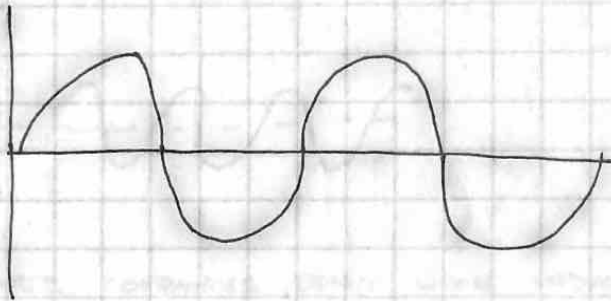
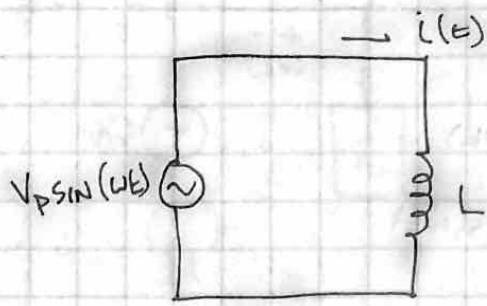
$$P(t) = \frac{V_p^2}{R} \sin^2(\omega t) \leftarrow \text{INSTANTANEOUS POWER}$$



$$P(t) = \frac{V_p^2}{R} \frac{1}{2} [1 - \cos(2\omega t)] = P(t) = \frac{V_p^2}{R} \sin^2(\omega t)$$

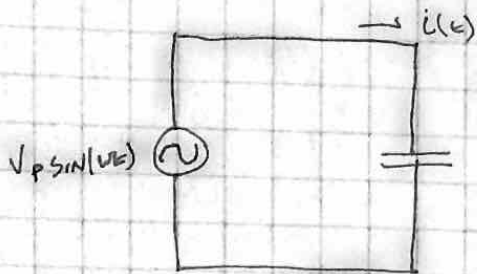
$$P_{AVE} = \frac{1}{T} \int_0^T P(t) dt \Rightarrow P_{AVE} = \frac{1}{T} \int_0^T \frac{V_p^2}{R} \frac{1}{2} [1 - \cos(2\omega t)] dt$$

$$\frac{V_p^2}{2RT} \left[\int_0^T 1 dt - \int_0^T \cos(2\omega t) dt \right] \Rightarrow \dots \Rightarrow \frac{V_p^2}{2R}$$



$P_{AVE} = 0$ INDUCTIVE LOAD

$$P(t) = v(t)i(t) = \frac{-V_p^2}{\omega L} \left(\frac{1}{2} \sin(2\omega t) \right)$$



$$v(t) = V_p \sin(\omega t)$$

$$i(t) = \omega V_p L \cos(\omega t)$$

TRIG IDENTITIES:

$$\sin^2 \theta = \frac{1}{2} [1 - \cos(2\theta)]$$

$$\sin \theta \cos \theta = \frac{1}{2} \sin(2\theta)$$

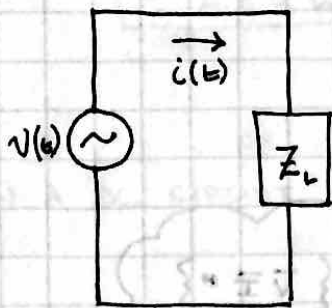
$$\cos \alpha + \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

ELI THE ICE MAN

* ELI: VOLTAGE (E) LEADS CURRENT (I) IN AN INDUCTIVE CIRCUIT (L)

* ICE: (I) CURRENT LEADS VOLTAGE (E) IN A CAPACITIVE CIRCUIT.

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$$i(t) = I_p \cos(\omega t + \phi_i)$$

$$v(t) = V_p \cos(\omega t + \phi_v)$$

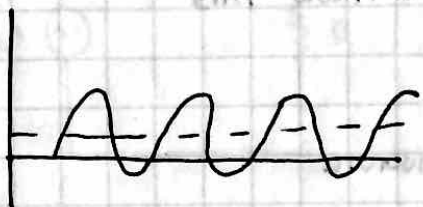
$$p(t) = I_p V_p \cos(\omega t + \phi_v - \phi_i)$$

$$= V_p I_p \left[\cos(\phi_v - \phi_i) + \cos(2\omega t + \phi_v + \phi_i) \right]$$

$\frac{1}{2}$

NO DEPENDENCE ON TIME = DIRECT CURRENT

ALSO, THIS IS THE POWER FACTOR



POWER COMPANIES DON'T LIKE INDUCTIVE/CAPACITIVE LOADS

RESISTOR:

$$\bar{V} = \bar{I} R \Rightarrow \frac{\bar{V}}{\bar{I}} = R = \frac{V_p}{I_p} \frac{\angle \phi_v}{\angle \phi_i} = \frac{V_p}{I_p} \angle \phi_v - \phi_i$$

HAS TO = 0 FOR RESISTOR

$$\cos(0) = 1 \quad \leftarrow \text{POWER FACTOR}$$

CAPACITOR:

$$\bar{V} = \bar{I} Z \Rightarrow Z = \frac{\bar{V}}{\bar{I}} = \frac{V_p}{I_p} \angle \phi_v - \phi_i = \frac{1}{j\omega C} = \frac{1}{\omega C} \angle -90$$

$$\cos(-90) = 0$$

POWER FACTOR

"LEADING"

= CAPACITIVE LOAD

INDUCTOR:

$$\frac{V_p}{I_p} \angle \phi_v - \phi_i = j\omega L = \omega L \angle +90$$

$$PF = 0$$

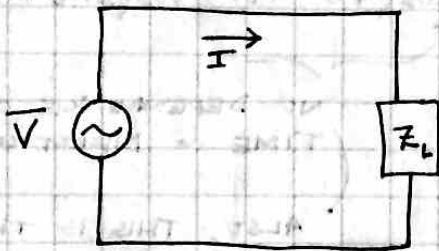
"LAGGING"

= INDUCTIVE LOAD

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GENERAL LOAD:

$$\bar{Z} = |Z| \angle \phi_z \Rightarrow PF = \cos(\angle \phi_z)$$



$$P_{AVE} = \frac{1}{2} \operatorname{Re} \{ \bar{V} \bar{I}^* \}$$

KNOW THIS

$$V = V \angle \phi_v$$

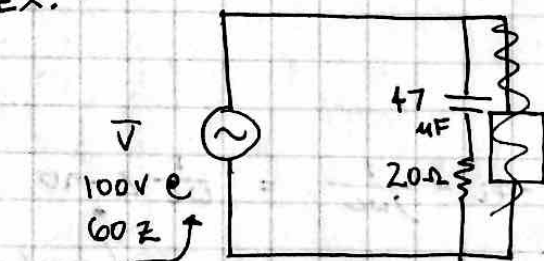
$$I = I_p \angle \phi_i \quad \leftarrow \text{COMPLEX CONJUGATE}$$

$$I^* = I_p \angle -\phi_i$$

$$\bar{V} \bar{I}^* = V_p I_p \angle \phi_v - \phi_i \Rightarrow \operatorname{Re} \{ \bar{V} \bar{I}^* \} = V_p I_p \cos(\phi_v - \phi_i)$$

$$P_{AVE} = \frac{1}{2} \operatorname{Re} \{ I Z I^* \} = \frac{1}{2} \operatorname{Re} \{ |I|^2 Z \} = \frac{1}{2} |I|^2 \operatorname{Re} \{ Z \}$$

EX.



$$I = 100V \angle 0$$

$$P_{AVE} = \frac{1}{2} \operatorname{Re} \{ \bar{V} \bar{I} \}$$

$$I = \frac{\bar{V}}{Z} \quad \text{WHERE } Z = 20\Omega + \frac{1}{j\omega C}$$

$$= 20\Omega - j56\Omega$$

$$= 60\Omega \angle -70.5^\circ$$

$$I = \frac{100V \angle 0}{60\Omega \angle -70.5} = 1.67A \angle 70.5^\circ$$

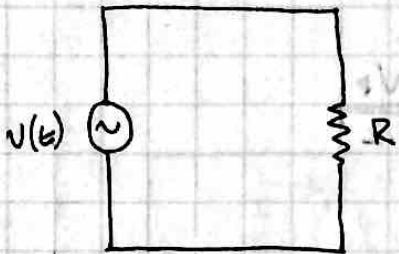
$$P_{AVE} = \frac{1}{2} \operatorname{Re} \{ \bar{V} \bar{I}^* \}$$

$$P_{AVE} = \frac{1}{2} \operatorname{Re} \{ (100 \angle 0)(1.67 \angle -70.5) \}$$

$$= \frac{1}{2} \operatorname{Re} \{ 1.67 \angle -70.5^\circ \}$$

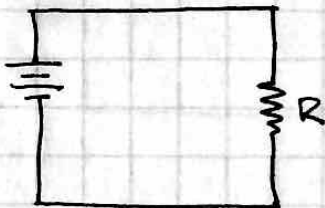
$$= 27.8 \text{ W}$$

NOW A DC CIRCUIT:



$$P(t) = v(t) i(t) = \frac{v(t)^2}{R}$$

$$P_{\text{AVE}} = \frac{1}{T} \int_0^T \frac{v^2(t)}{R} dt$$



$$P_{\text{AVE}} = \frac{V_{DC}^2}{R}$$

$$P_{\text{AVE}} = \frac{1}{R} \frac{1}{T} \left(\int_0^T v^2(t) dt \right) \Rightarrow P_{\text{AVE}} = \frac{V_{\text{EFF}}^2}{R}$$

$$V_{\text{EFF}} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

↑ ROOT
 ↑ MEAN
 ↑ SQUARED

$$V_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$$

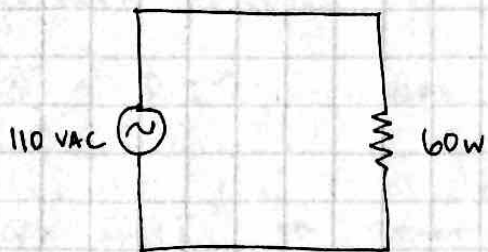
$$v(t) = V_p \sin(\omega t)$$

$$V_{RMS} = \sqrt{\frac{1}{T} \int_0^T V_p^2 \sin^2(\omega t) dt}$$

$$= \sqrt{\frac{V_p^2}{T} \int_0^T \frac{1}{2} (1 - \cos^2(\omega t)) dt}$$

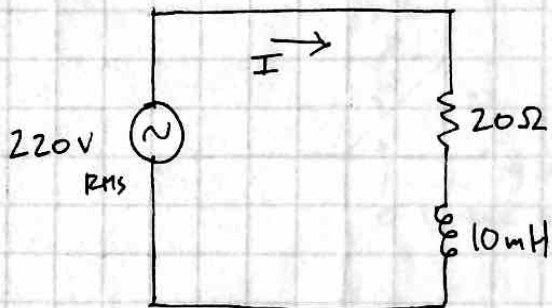
$$= \sqrt{\frac{V_p^2}{T} \int_0^T \frac{1}{2} dt} = \sqrt{\frac{V_p^2}{2}} = \frac{V_p}{\sqrt{2}}$$

$$V_{RMS} = \frac{V_p}{\sqrt{2}}$$



$$P = \frac{(V_{RMS})^2}{R}$$

$$R = 201 \Omega$$



$$Z = 20 \Omega + j\omega L$$

$$= 20 + j377 \Omega$$

$$= 20.35 \Omega \angle 10.7^\circ$$

$$I_{RMS} = \frac{220V}{20.75 \Omega \angle 10.7^\circ} = 10.81 A \angle -10.7^\circ$$

$$P = (220V_{RMS})(10.81) = 2378 W$$

$$P_{AVE} = \text{Re}\{\sqrt{V} I^*\}$$

$$P_{AVE} = (220 \angle 0^\circ)(10.81 \angle 10.7^\circ)$$

$$= 2337 W$$

$$* S = VI$$

$$P = |S| \underbrace{\cos(\phi_v - \phi_i)}_{PF}$$

$$\frac{P}{S} = P.F.$$

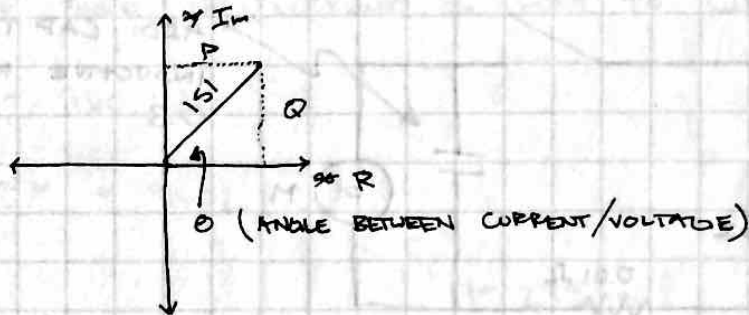
FEBRUARY 7, 2012

COMPLEX POWER:

$$\bar{S} = \bar{V} I^* \quad \text{WHERE } P = \operatorname{Re}\{\bar{S}\} \quad \leftarrow \text{REAL POWER}$$

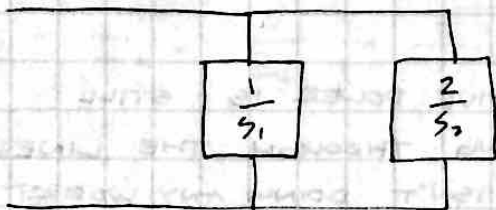
$$Q = \operatorname{Im}\{\bar{S}\} \Rightarrow \text{REACTIVE POWER}$$

$$|\bar{S}| = S = VI^* \Rightarrow \text{"APPARENT POWER"}$$



$$\text{PF} = \frac{P}{S} = \cos \phi$$

LAGGING = INDUCTIVE LOADS



$$\bar{S}_{\text{TOT}} = \bar{S}_1 + \bar{S}_2$$

$$P_{\text{TOT}} = P_1 + P_2$$

$$Q_{\text{TOT}} = Q_1 + Q_2$$

UNITS:

$$\bar{S} = \text{VA (INSTEAD OF WATTS)}$$

$$P = \operatorname{Re}\{\bar{S}\} \text{ W (WATTS)}$$

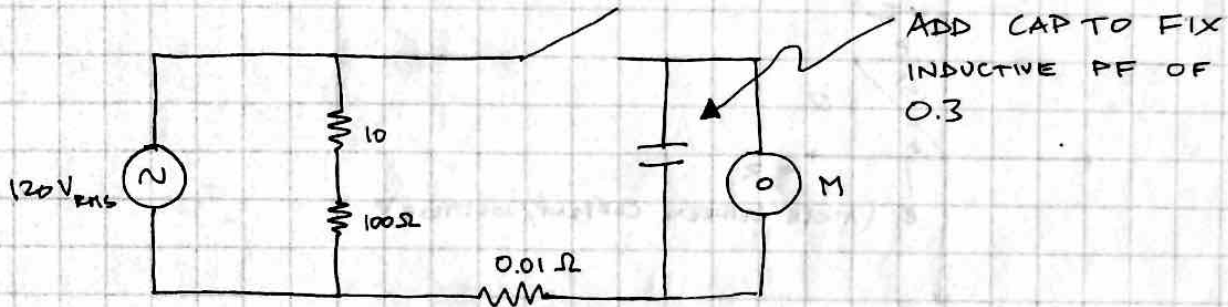
$$Q = \operatorname{Im}\{\bar{S}\} \text{ (VAR)} \quad \leftarrow \text{NOT "REAL POWER"}$$

MOTOR NAMEPLATE :

$$\frac{1}{3} \text{ Hp} = 745.7 \text{ W}$$

* RATED HP RATINGS = MECHANICAL OUTPUT POWER, NOT ELECTRICAL INPUT.

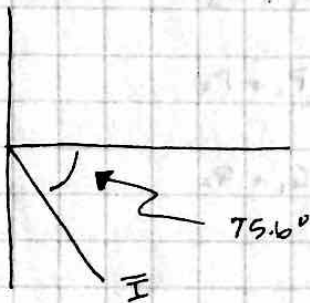
* S.F. "SERVICE FACTOR" TIMES YOU CAN OVERLOAD THE MOTOR



$$V = 120V \angle 0^\circ \quad I = \frac{11.2A}{\sqrt{2}} = 8A_{RMS}$$

$$\phi_v - \phi_i = \frac{3.5ms}{16.66\mu s} = \text{PHASE DIFFERENCE BY } 75.6^\circ$$

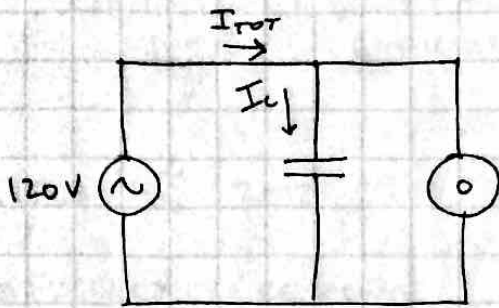
$$A = 8A_{RMS} \angle -75.6^\circ$$



REACTIVE POWER IS STILL FLOWING THROUGH THE LINES BUT ISN'T DOING ANY WORK

PF = 0.3 LAGGING

FEBRUARY 9, 2012



$$\bar{V} = 120V \angle 0^\circ$$

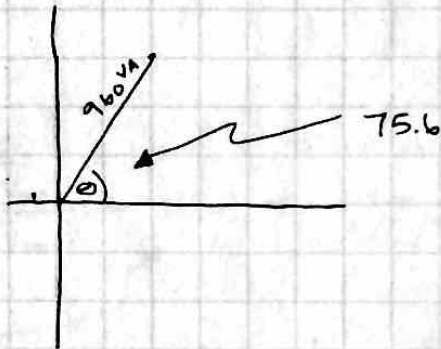
$$\bar{I}_L = 8A \angle -75^\circ$$

$$= 2 - j7.75A$$

GCFI: MEASURES CURRENT OUT OF ONE PLUG INTO ANOTHER TO MAKE SURE NO CURRENT IS LOST TO GROUND.

$$S = VI = 960 \text{ VA}$$

$$\bar{S} = \bar{V} \bar{I}^* = 960 \angle 75.6^\circ$$



$$P = 239 \text{ W}$$

$$Q = 930 \text{ VAR}$$

$$\text{PF} = 0.3$$

$$I_C = \frac{V}{Z_C} = \frac{\bar{V}}{1/j\omega C} = V_j \omega C = 7.75$$

TEST:

CHAPTERS 9, 10, 11 FAIR GAME, NOTHING W/ OP-AMP
 ~3 QUESTIONS. KNOW QUESTIONS FROM
 THE BACK OF BOOK.

REVIEW:

- Z NORTON / THEVENIN
- CURRENT / VOLTAGE DIVISION
- ALL EQUATIONS

$$S = I_{RMS}^2 Z = \frac{V_{RMS}^2}{Z^*} = V_{RMS} I_{RMS}^*$$

$$P_{MAX} = \frac{V_{TH}^2}{8R_{TH}}$$

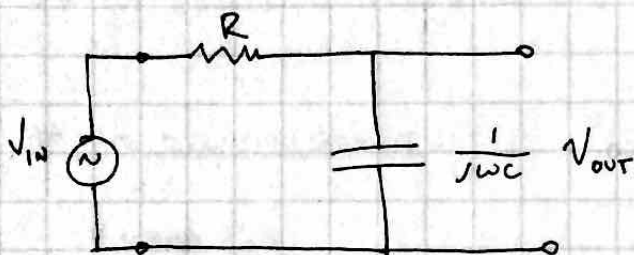
$$P_{AVE} = \frac{1}{2} \text{Re}\{V I^*\}$$

FEB, 9TH, 2012

$V = 480\text{ V}$, $\text{PF} = 93\%$
 $\text{HP} = 5$, $\text{EFFICIENCY} = 81\%$

FEB 16TH, 2012

* FREQUENCY RESPONSE



$$\bar{V}_{\text{OUT}} = V_{\text{IN}} \left(\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} \right)$$

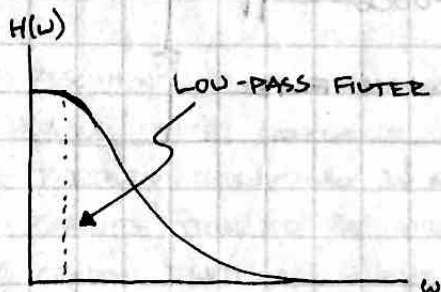
LOW-PASS FILTER

$$\frac{\bar{V}_{\text{OUT}}}{V_{\text{IN}}} = H(\omega) = \frac{1}{1 + j\omega RC}$$

$$\frac{1}{RC} = \omega_0$$

UNITS 'SECONDS'

TRANSFER FUNCTION



$$|H(\omega)| = \sqrt{\left(\frac{1}{1 + j\omega/\omega_0} \right)^2 + \left(\frac{1}{1 - j\omega/\omega_0} \right)^2}$$

$$= \sqrt{\frac{1}{1 + \omega^2/\omega_0^2}}$$

FOR $\omega \ll \omega_0$, $|H(\omega)| \Rightarrow 1$

FOR $\omega_0 \gg \omega$, $|H(\omega)| \Rightarrow \frac{\omega_0}{\omega} = 0$

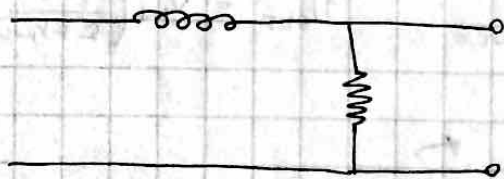
FOR $\omega = \omega_0$, $|H(\omega)| \Rightarrow \frac{1}{\sqrt{2}}$

* PHASE RESPONSE

$$H(\omega) = \frac{1}{1 + j\omega/\omega_0} \frac{(1 - j\omega/\omega_0)}{1 - j\omega/\omega_0} = \frac{1 - j\omega/\omega_0}{1 + \omega^2/\omega_0^2} \Rightarrow$$

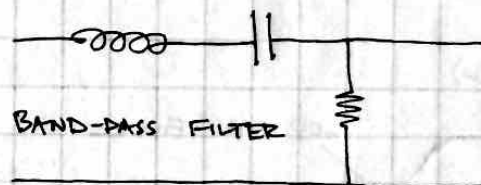
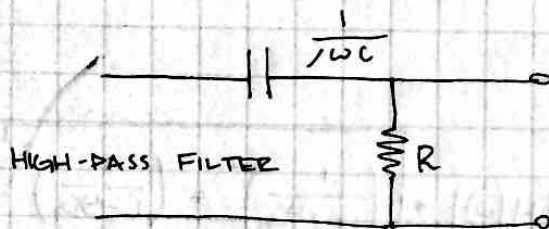
$$H(\omega) = \frac{1}{1 + \omega^2/\omega_0^2} - j \frac{\omega/\omega_0}{1 + \omega^2/\omega_0^2}, \quad \angle H(\omega) = \text{TAN}^{-1}\left(\frac{-\omega}{\omega_0}\right)$$

SIMILAR TO :



$$\text{dB} \approx \log\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \Rightarrow 10 \log\left(\frac{V_{\text{out}}^2}{V_{\text{in}}^2}\right) = 20 \log\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$$

$$\text{dB}_{\text{m}} = \text{"DECIBEL w/ RESPECT TO A MILLIVOLT"} = 20 \log \frac{V_{\text{out}}}{V_{\text{in}}}$$



CHAPTER 14 READING NOTES

* FREQUENCY RESPONSE: THE FREQUENCY RESPONSE OF A CIRCUIT IS THE VARIATION IN ITS BEHAVIOR WITH CHANGE IN SIGNAL FREQUENCY.

* TRANSFER FUNCTION: THE TRANSFER FUNCTION $H(\omega)$ IS THE FREQUENCY DEPENDENT RATIO OF PHASOR OUTPUT $Y(\omega)$ TO PHASOR INPUT $X(\omega)$.

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} \Rightarrow H(\omega) = \frac{N(\omega)}{D(\omega)}$$

* THE DECIBEL SCALE:

HISTORICALLY,

$$G = \text{NUMBER OF BELLS} = \log_{10} \left(\frac{P_2}{P_1} \right)$$

$$G_{dB} = 10 \log_{10} \left(\frac{P_2}{P_1} \right)$$

AND

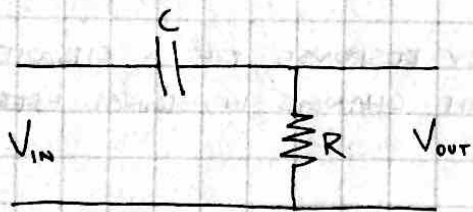
$$G = 20 \log_{10} \left(\frac{V_2}{V_1} \right)$$

* BODE PLOTS: A BODE PLOT IS A SEMI-LOG PLOT OF THE MAGNITUDE (IN DECIBELS) AND THE PHASE (IN DEGREES) OF A TRANSFER FUNCTION VERSUS FREQUENCY.

* RESONANCE: RESONANCE IS A CONDITION OF AN RLC CIRCUIT IN WHICH THE CAPACITIVE AND INDUCTIVE REACTANCES ARE EQUAL, THEREBY RESULTING IN A PURELY RESISTIVE IMPEDENCE. THEIR TRANSFER CIRCUITS TEND TO BE HIGHLY FREQUENCY SELECTIVE, SO THEY ARE OFTEN USED AS FILTER CIRCUITS.

* FILTERS: A FILTER IS A CIRCUIT DESIGNED TO PASS SIGNALS WITH CERTAIN FREQUENCIES, AND REJECT OR ATTENUATE OTHERS.

* HIGH-PASS FILTER:

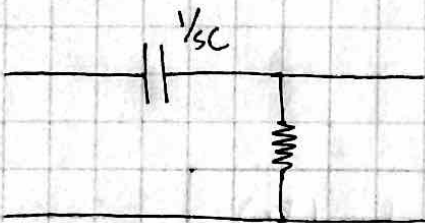


$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC}$$

$$\omega / \omega_0 = \frac{1}{RC}$$

$$H(\omega) = \frac{j\omega/\omega_0}{1 + j\omega/\omega_0}$$

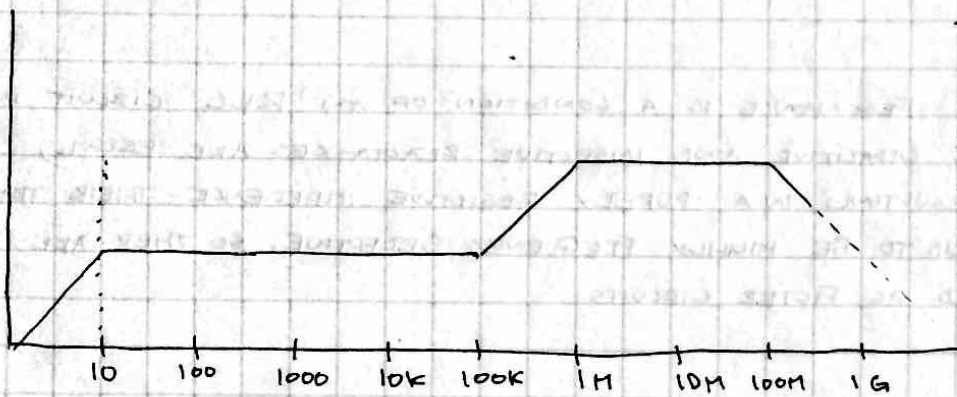
IN LAPLACE DOMAIN:



$$s = j\omega$$

IN GENERAL, ANY TRANSFER FUNCTION CAN BE WRITTEN IN A FORM WITH "ZEROS" ON TOP, AND POLES ON "BOTTOM"

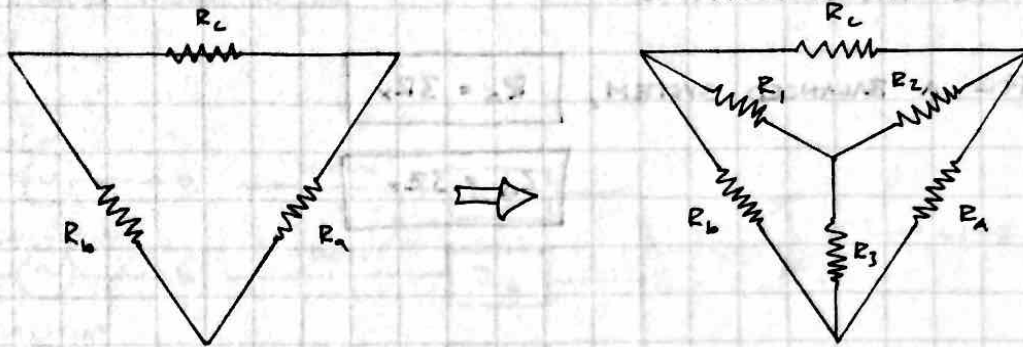
$$H(s) = \frac{As(1 + s/2\pi 10\text{kHz})}{(1 + s/2\pi 10\text{Hz})(1 + s/2\pi 100\text{kHz})(1 + s/2\pi 100\text{MHz})}$$



F	P	
0	Z	+20
10	P	0
10K	Z	+20
100K	P	0
100M	P	-20

NEXT STEP IS TO FIND SCALE

* DELTA TO WYE CONVERSION:



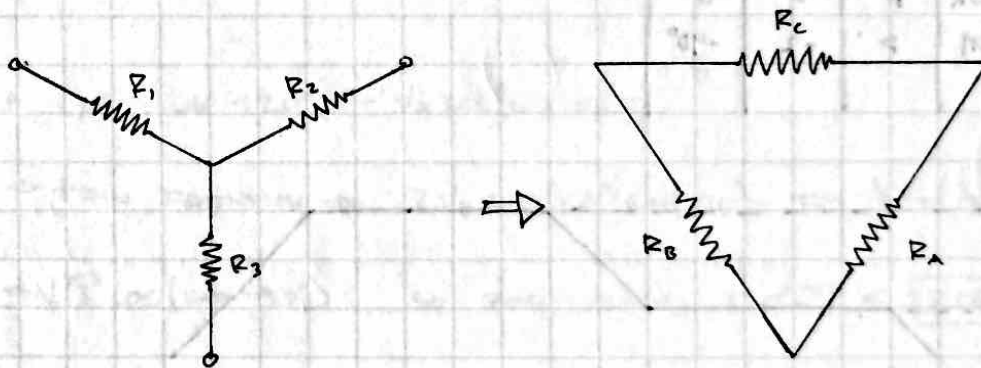
EACH RESISTOR IN THE WYE NETWORK IS THE PRODUCT OF THE TWO ADJACENT Δ BRANCHES, DIVIDED BY THE SUM OF THE THREE Δ RESISTORS

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

* WYE TO DELTA CONVERSIONS:



EACH RESISTOR IN THE DELTA NETWORK IS THE PRODUCT OF ALL POSSIBLE WYE RESISTORS TAKEN TWO AT A TIME, DIVIDED BY THE OPPOSITE Y RESISTOR.

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

2/28/2012

DELTA-WYE TRANSFORMATIONS:

FOR A BALANCED SYSTEM,

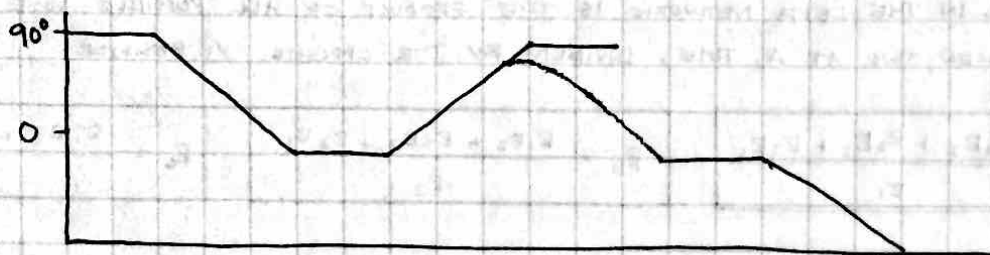
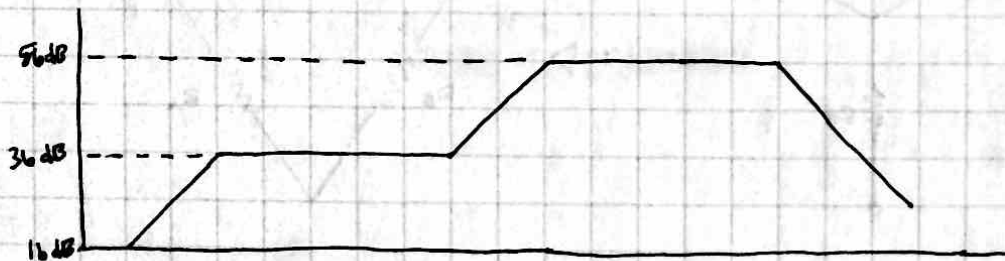
$$R_{\Delta} = 3R_Y$$

$$Z_{\Delta} = 3Z_Y$$

TRANSFER FUNCTION:

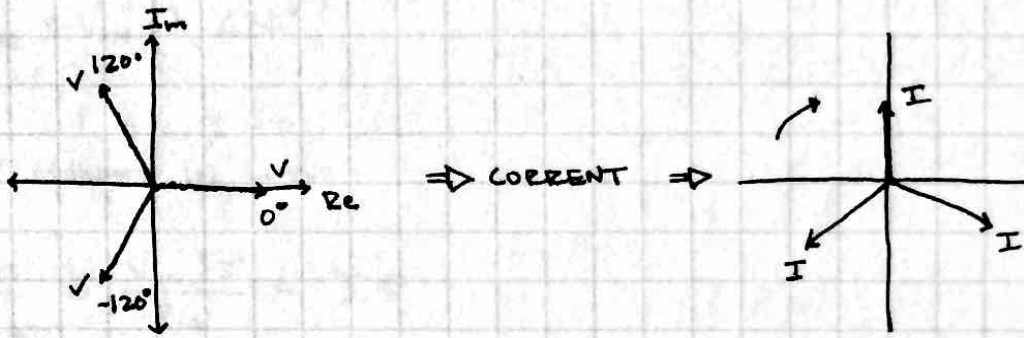
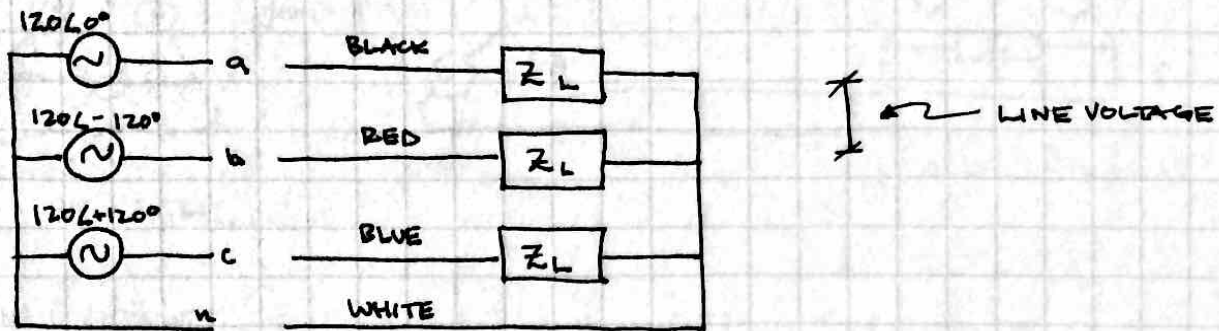
$$H(s) = \frac{Ks(1 + s/2\pi 10K)}{(1 + s/2\pi 10)(1 + s/2\pi 100K)(1 + s/2\pi 100M)}$$

f	D/Z	SLOPE	ϕ
0	Z	+20	+90°
10	P	0	0
10K	Z	+20	+90°
100K	P	0	0
100M	P	-20	-90°



2/28/2012

* THREE-PHASE POWER

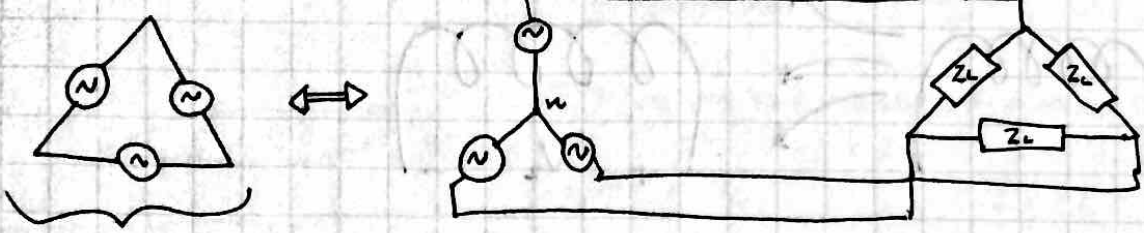


$$V_{BC} = V_p \cos(\omega t - 120^\circ) - V_p \cos(\omega t + 120^\circ)$$

$$\Rightarrow \text{TRIG IDENTITY} \Rightarrow 2V_p \sin(\omega t) \sin(120) \Rightarrow 2V_p \left(\frac{\sqrt{3}}{2}\right) \cos(\omega t - 90^\circ)$$

$$= \sqrt{3} V_p \cos(\omega t - 90^\circ), \text{ so FOR } 120V, 120 \cdot \sqrt{3} = \boxed{208V}$$

3/6/2012



NO NEUTRAL

LINE VOLTAGE :

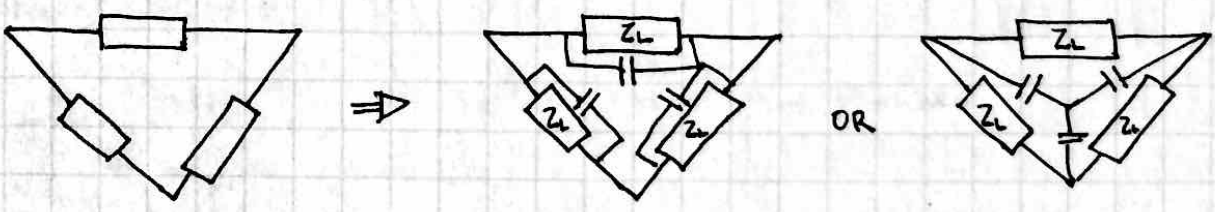
$$V_{AB} = V_{AN} \sqrt{3} \angle 30^\circ$$

PHASE CURRENT IN LOAD :

$$I_{\phi} = \frac{V_{AB}}{Z_{\Delta}} = \frac{V_{AN} \sqrt{3}}{Z_{\Delta}} = I_{\phi}$$

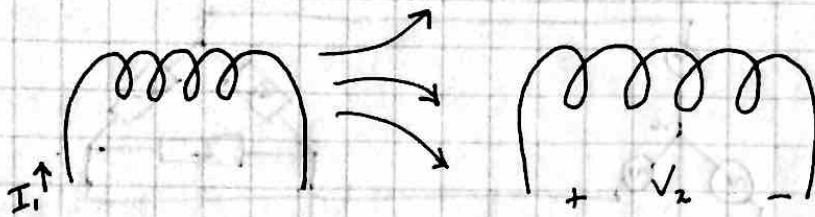
POWER IN LOAD : $|I_{LINE}|^2 (R_p)(3) =$ ← ONLY RESISTIVE COMPONENT OF Z_Y

POWER FACTOR CORRECTION CAN BE USED ON 3-PHASE SYSTEMS AS WELL



FINAL AT 7:30AM AT GIVE AWD!

3/6/2012



FARADAY'S LAW:

$$V_2 = \frac{d\phi}{dt} N_2$$

$$\phi = BA \mu \epsilon \mu_0 I_1, \quad \phi \propto I_1$$

$$V_2 = \underbrace{N_2 k}_{\text{CONSTANT}} \frac{dI_1}{dt}$$

$$V_2 = L \frac{dI_1}{dt}, \text{ so } \underbrace{N_2 k}_{\rightarrow L} \Rightarrow V_2 = M_{21} \frac{dI_1}{dt}$$

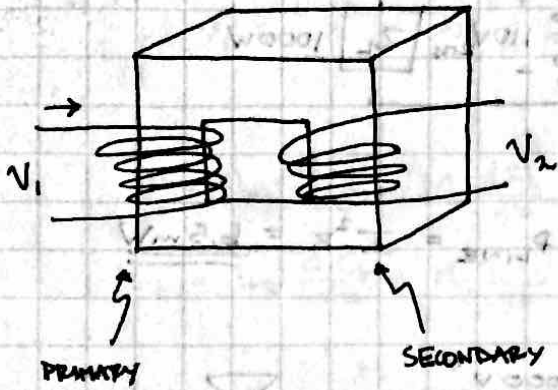
$$M_{12} = M_{21}$$

"MECHANICAL COUPLING"



3/8/2012

TRANSFORMER



IRON CHANNELS MAGNETIC FLUX

$$\Phi_1 = \Phi_2$$

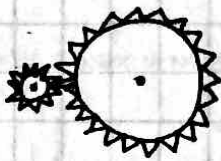
$$V_1 = \frac{d\Phi_1}{dt} N_1, \quad V_2 = \frac{d\Phi_2}{dt} N_2$$

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} \Rightarrow P_2 = P_1 = V_2 I_2 = V_1 I_1$$

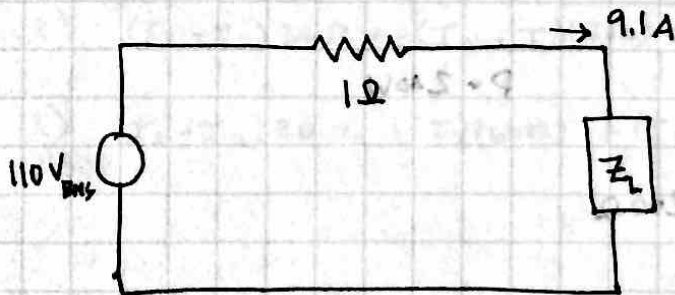
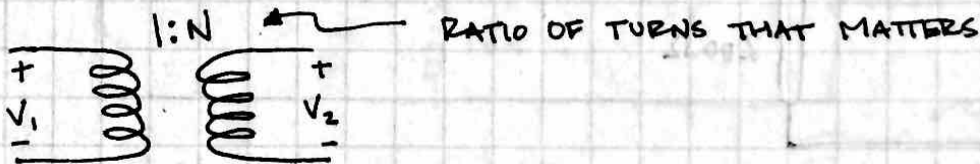
$$\frac{I_2}{I_1} = \frac{N_1}{N_2}$$

MUST BE A TIME-VARYING FIELD
(AC ONLY) BECAUSE POWER DISTRIBUTION
IS AC AROUND COUNTRY.

ANALOGOUS TO A GEARBOX:

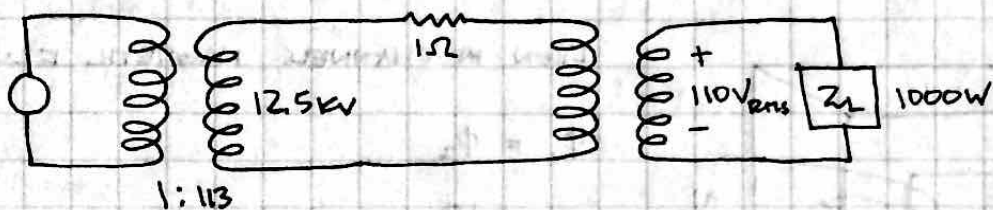


CIRCUIT DIAGRAM:



$$P_{LINE} = (9.1)^2 (1\Omega) = \underline{82W}$$

INSTEAD, TRANSFORM:



$$I_{\text{LINE}} = \frac{9.1A}{113} = 80\text{mA}, \quad P_{\text{LINE}} = I^2 R = \underline{6.5\text{mW}}$$

"EVERY LITTLE INSULATOR IS ABOUT 15,000V"



$$\bar{V}_2 = N\bar{V}_1, \quad I_2 = \frac{1}{N}\bar{I}_1$$

$$N\bar{V}_1 = \frac{1}{N}\bar{I}_1 Z_L$$

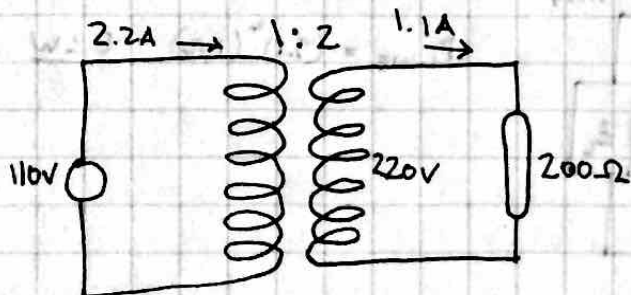
HIGH-IMPEDENCE DUE TO TRANSFORMER

HOT DOG:

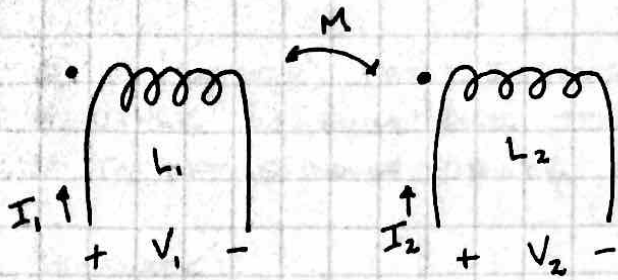


~40s

INSTEAD:



$$P = 240W$$

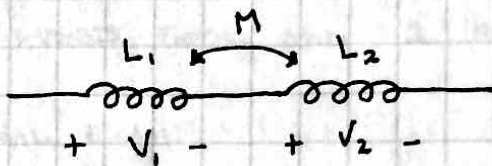


$$V_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$\bar{V}_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

$$\bar{V}_2 = j\omega L_2 I_2 + j\omega M I_1$$



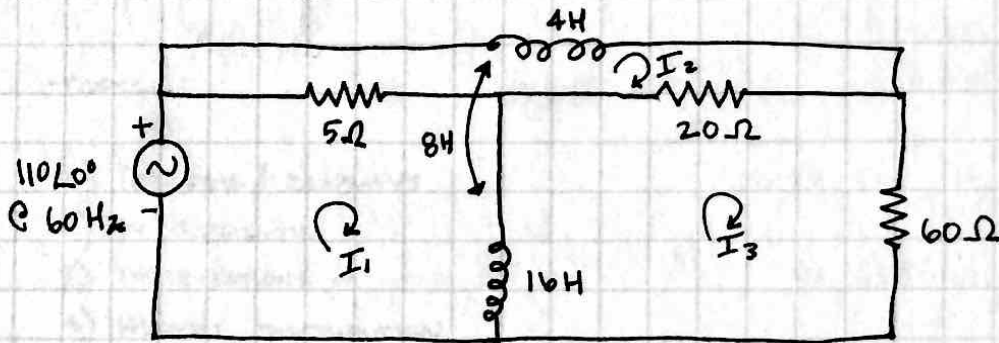
$$V_1 = j\omega L_1 I + j\omega M I$$

$$V_2 = j\omega L_2 I + j\omega M I$$

$$\bar{V}_1 + \bar{V}_2 = V_{\text{TOTAL}} = j\omega (L_1 + L_2 + 2M) I$$

$$L_{\text{EQ}} = L_1 + L_2 + 2M$$

MUTUAL INDUCTANCE EX:



* DOTS ONLY COME IN FOR MUTUAL INDUCTANCE *

FROM MUTUAL IND.

$$1.) -110\angle 0^\circ + (I_1 - I_2)5\Omega + (I_1 - I_3)j\omega 16H + \bar{I}_2(j\omega 8H)$$

$$2.) (I_2 - I_3)20\Omega + (I_2 - I_1)5\Omega + I_2(j\omega 4H) + (I_1 - I_3)(j\omega 8H)$$

$$3.) (I_3 - I_2)20\Omega + I_3(60\Omega) + (I_3 - I_1)j\omega 16H - I_2 j\omega 8H$$