HEAT TRANSFER 10/1/2013

CONSERVATION OF ENERGY

DIFFERENCE BETWEEN ADVECTION / CONVECTION

$$E = \frac{1}{2}mV^2 + mu + mgz$$

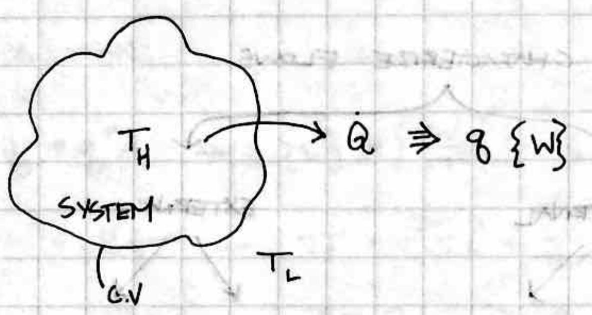
* CLOSED SYSTEM: $Q, W \rightarrow \frac{dE}{dt} = \dot{Q}_{NET} - \dot{W}_{NET}$

* OPEN SYSTEM: $\dot{Q}, \dot{W}, \underbrace{\dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{ADVECTED}}$

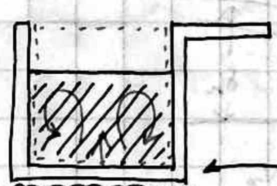
$$\rightarrow \frac{dE}{dt} = \dot{Q}_{NET} - \dot{W}_{NET} + \sum \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right) - \sum \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)$$

HEAT TRANSFER:

$\dot{Q} \Rrightarrow q$ ← USING THIS NOTATION

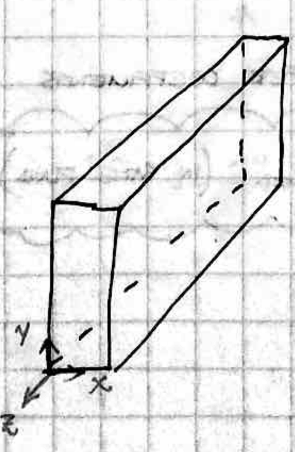


CONDUCTION: HT DUE TO LATTICE VIBRATIONS + ELECTRON MOBILITY (IN CONDUCTOR)



CONDUCTION: ACROSS SOLID THRU STATIC FLUID

← HT STARTS W/ CONDUCTION. AS H_2O TEMP ↑, ρ↓ & H_2O MOVES. → CONVECTION OCCURS. → DIFFUSSION + ADVECTION



FOURIER'S LAW

$$q = -KA\nabla T$$

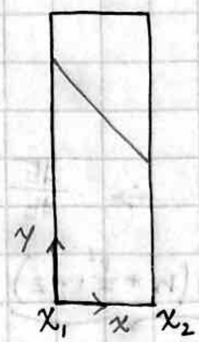
$$\nabla T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$$

K = THERMAL CONDUCTIVITY (UNITS $\frac{W}{m \cdot K}$)

A = AREA NORMAL TO TRANSPORT OF HEAT.

$$q'' = -k\frac{\partial T}{\partial x}$$

$$q = q''A \rightarrow q'' = \frac{q}{A}$$



$$\frac{T_2 - T_1}{x_2 - x_1} \quad \frac{(-)}{(+)}$$

ALIGN @ X+ IN DIRECTION

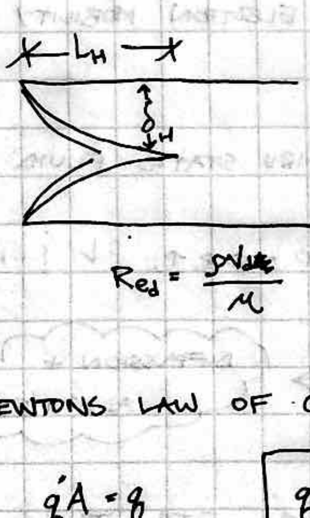
*CONVECTION:

DIFFUSSION + ADVECTION ⟶ FLUID MECHANICS

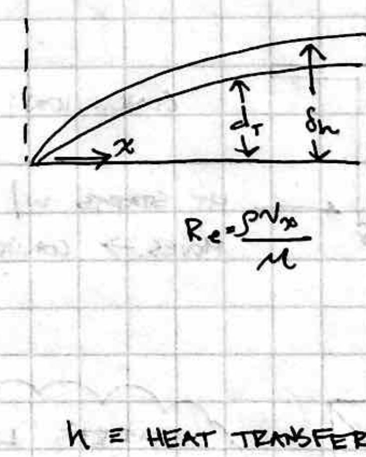
$$\boxed{\begin{aligned} \tau_w &= \mu\frac{\partial u}{\partial y} \\ \tau_w A &= F_{FRICTION} \end{aligned}}$$

CHARACTERIZE FLOWS

- INTERNAL
  - LAM
  - TURB
- EXTERNAL
  - LAM
  - TURB



$$Re_d = \frac{\rho V d}{\mu}$$



$$Re = \frac{\rho V x}{\mu}$$

*NEWTONS LAW OF COOLING

$$\dot{q}A = q \qquad \boxed{q = hA(T_H - T_c)}$$

h ≡ HEAT TRANSFER COEFFICIENTS

$h_{NATURAL} < h_{FORCED}$ (IN SAME FLUID)

WHY???

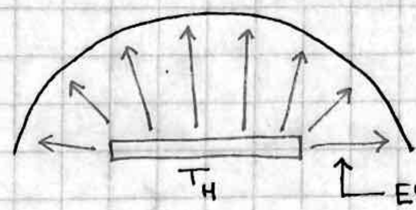
10/1/2013

RADIATION: AN "EXCHANGE"

$$q = h_r A (T_H - T_c) \qquad q = \varepsilon \sigma A (T_H^4 - T_c^4)$$

h_r → RADIATION HTC

ε → EMISSIVITY



"IF IT REACHES YOU, ITS IRRADIATION"

10/3/2013

CONDUCTION:

$$q = q'' A \rightarrow \{W\} = \left\{\frac{W}{m^2}\right\}\{m^2\}$$

$$q'' = -K \nabla T \rightarrow \nabla T = \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k} \rightarrow \begin{matrix} x & y & z \\ r & \phi & z \end{matrix}$$

$$\nabla T = \frac{\partial T}{\partial r}\hat{i} + \frac{1}{r}\frac{\partial T}{\partial \phi}\hat{j} + \frac{\partial T}{\partial z}\hat{k}$$

$$q'' = q_x''\hat{i} + q_y''\hat{j} + q_z''\hat{k} \rightarrow q = q_x'' A_x \vec{i} + q_y'' A_y \hat{j} + q_z'' A_z \hat{k}$$

THE HEAT FLUX — THE HEAT ~~FLOW~~ TRANSPORT

→ GLOBAL INTEGRAL BALANCE

$$\frac{dE}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{GEN}$$

$\dot{E}_{in}$: $q, \dot{W}, \dot{m}$

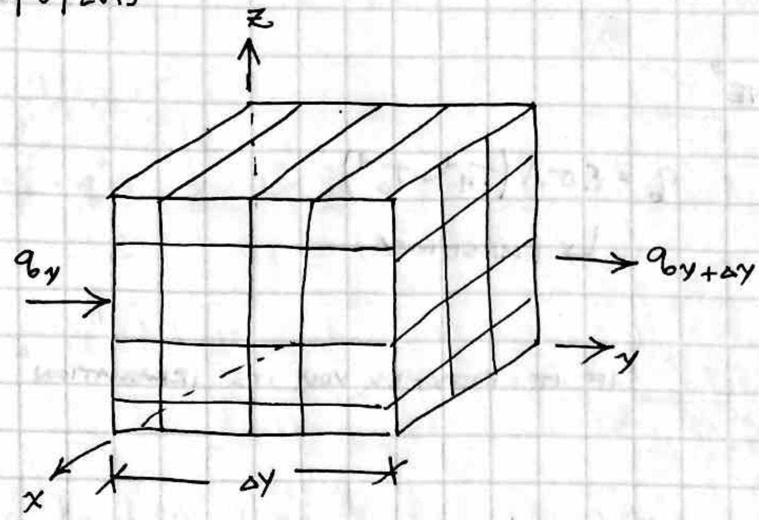
$\dot{E}_{GEN}$: NEW; NEW TO USE

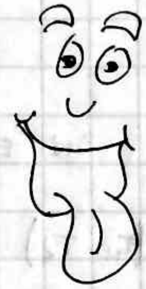
$$\frac{dE}{dt} = q_{in} - q_{out} + q\forall$$

q: $\frac{W}{m^3}$

$$\boxed{q' = \frac{W}{m} \qquad q'' = \frac{W}{m^2} \qquad q''' = \frac{W}{m^3}}$$

10/3/2013





DR. PENCE

$$q_{y+\Delta y} = q_y + \frac{\partial q_y}{\partial y}\Delta y$$

$$q_{x+\Delta x} = q_x + \frac{\partial q_x}{\partial x}\Delta x \quad \ldots$$

$$\frac{dE}{dt} = \cancel{q_x} - \left(\cancel{q_x} + \frac{\partial q_x}{\partial x}\Delta x\right) + \cancel{q_y} - \left(\cancel{q_y} + \frac{\partial q_y}{\partial y}\Delta y\right) +$$

$$+ \cancel{q_z} - \left(\cancel{q_z} + \frac{\partial q_x}{\partial z}\right) + \dot{q}(\Delta x \Delta y \Delta z)$$

$$\frac{dE}{dt} = -\frac{\partial}{\partial x}\left(-KA_x\frac{\partial T}{\partial x}\right)\Delta x + \ldots$$

$$= \frac{\partial}{\partial x}\left(K\Delta y\Delta z\frac{\partial T}{\partial x}\right)\Delta x \longrightarrow \cancel{\frac{\partial}{\partial x}(K\Delta y\Delta z)}$$

$$\frac{\partial}{\partial x}\left(K\Delta y\Delta z\frac{\partial T}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)\Delta x\Delta y\Delta z + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)\Delta x\Delta y\Delta z + \dot{q}(\Delta x\Delta y\Delta z)$$

INSIGNIFICANT

$$E = U + PE + KE \longrightarrow E = mu + \cancel{mgz} + \cancel{\tfrac{1}{2}mv^2} \longrightarrow E = \rho[\Delta x\Delta y\Delta z]u$$

$$\frac{dE}{dt} = \rho[\Delta V]c\frac{dT}{dt} \longleftarrow \frac{\partial T}{\partial t}$$

→ HEAT DIFFUSSION EQN.

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \dot{q}$$

VOLUMETRIC HEAT CAPACITY

$$\alpha = \frac{K}{\rho c} = \frac{\text{TRANSPORT}}{\text{STORAGE}}$$

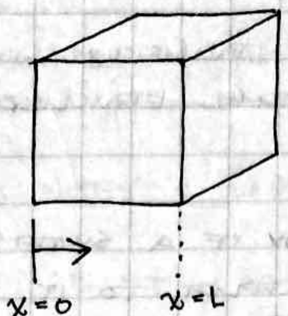
$$\boxed{Pr = \frac{\nu}{\alpha} \propto \frac{\delta_h}{\delta_t}}$$

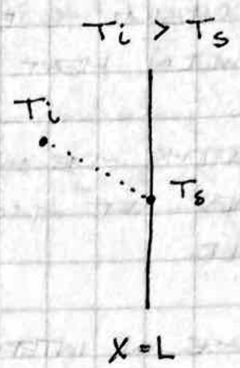
EES!

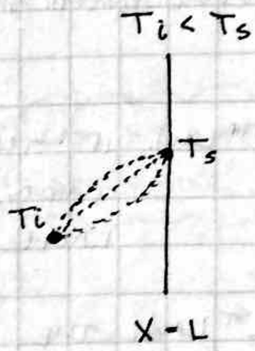
10/3/2013

→ BOUNDARY CONDITIONS ←

1. CONSTANT TEMP.







$T(x=L, t) = T_s$

2. CONSTANT HEAT FLUX $q''_x\Big|_{x=L} = -K \frac{\partial T}{\partial x}\Big|_{x=L}$

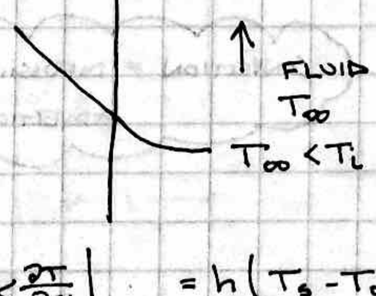
A.) $q'' = 0$ (INSULATED) → B.C. $\boxed{\frac{\partial T}{\partial x}(x=L, t) = 0}$

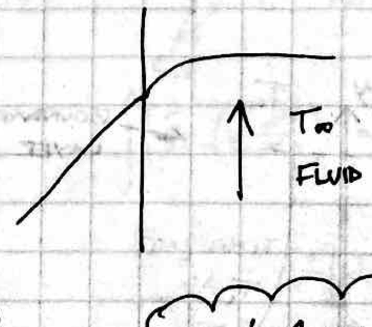
B.) $q'' \neq 0$

$\boxed{\begin{array}{l} q''(+) \ + \ x\text{-DIR} \\ q''(-) \ \text{IN} \ -x\text{-DIR} \end{array}}$

$q''_{x=L} = -K \frac{\partial T}{\partial x}\Big|_{x=L}$

3. CONVECTION:





$-K \frac{\partial T}{\partial x}\Big|_{x=L} = h(T_s - T_\infty)$

$-K \frac{\partial T}{\partial x}\Big|_{x=L} = h(T_s - T_\infty)$

$\varepsilon\sigma(T_s^4 - T_{\infty s}^4)$ ← RADIATION

B.P

10/5/2013 READING NOTES:

→REVIEW←

* THERMODYNAMICS: A PHYSICAL SCIENCE DEALING WITH THE RELATIONSHIPS BETWEEN HEAT AND OTHER FORMS OF ENERGY.

0TH LAW: IF TWO SYSTEMS ARE IN THERMAL EQUILLIBRIUM WITH A THIRD, THEY ARE ALSO IN THERMAL EQUILLIBRIUM WITH EACH OTHER.

1ST LAW: THE INCREASE IN INTERNAL ENERGY OF A SYSTEM IS EQUAL TO THE DIFFERENCE ~~OF~~ HEAT SUPPLIED TO IT MINUS THE WORK DONE BY IT → $\Delta U = Q - W$

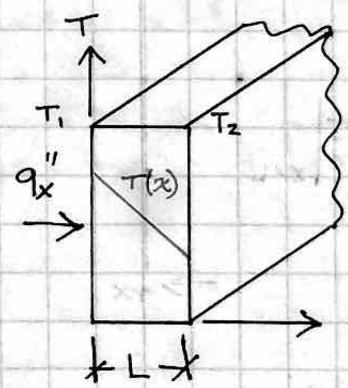
2ND LAW: HEAT CANNOT SPONTANEOUSLY FLOW FROM A COLD BODY TO A HOT BODY

→ READING NOTES ←

"HEAT TRANSFER IS THE TRANSIT OF THERMAL ENERGY DUE TO A SPACIAL TEMPERATURE DIFFERENCE"

* FOURIER'S LAW OF CONDUCTION:

$$q''_x = -K \frac{dT}{dx}$$

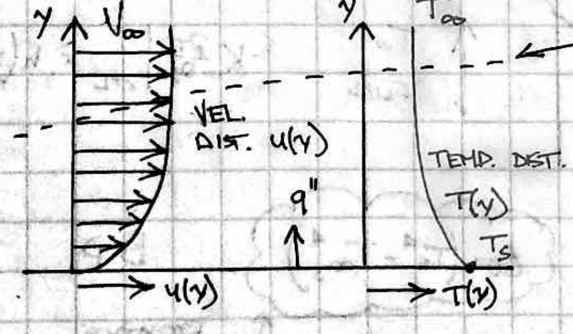


$T_1 > T_2$ = NEGATIVE $\frac{dT}{dx}$, BUT q''_x IS REQ. TO BE (+), SO "$-K$" IS NECC.

$\boxed{q''_x = \text{HEAT FLUX } \left\{\frac{W}{m^2}\right\}}$

$\boxed{K = \text{THERMAL CONDUCTIVITY } \left\{\frac{W}{m \cdot K}\right\}}$

* CONVECTION:



CONVECTION = DIFUSSION + ADVECTION

10|5|2013 READING NOTES:

REGARDLESS OF CONVECTION HT PROCESS:

$$\boxed{q'' = h(T_s - T_\infty)}$$ ← NEWTON'S LAW OF COOLING

$$\boxed{h = \text{CONVECTION HT COEFFICIENT } \left\{\frac{W}{m^2 \cdot K}\right\}}$$ ← CAN BE HARD TO DETERMINE

10|8|2013 NOTES IN-CLASS

3.1 - 3.4 1-D STEADY STATE CONDUCTION W/NO GENERATION

HDE:

$$\frac{dE}{dt} = \dot{E}_{IN} - \dot{E}_{OUT} + \dot{E}_{GEN}$$

CARTESIAN:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right)\vec{i} + \frac{\partial}{\partial y}\left(K \frac{\partial T}{\partial y}\right)\vec{j} + \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right)\vec{k} + \dot{q}$$

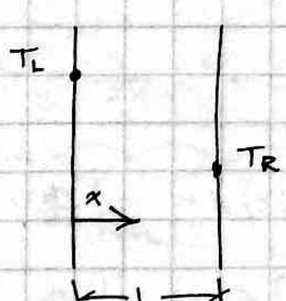
CYLINDRICAL:

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(Kr \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(K \frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right) + \dot{q}$$

* BOUNDARY CONDITIONS:

A. T = SPECIFIED
B. i. $q'' = 0$
   ii. $q'' \neq 0$
C. $q''_{COND} = q''_{CONV.}$

* EXAMPLE



$T_L > T_R$

$$\cancelto{S.S}{\rho c \frac{\partial T}{\partial t}} = \frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right) + \cancelto{1\text{-}D}{\frac{\partial}{\partial y}\left(K \frac{\partial T}{\partial y}\right)} + \cancelto{1\text{-}D}{\frac{\partial}{\partial z}\left(K \frac{\partial T}{\partial z}\right)} + \cancelto{NO}{\dot{q}}$$

$$\int\left[\frac{\partial}{\partial x}\left(K \frac{dT}{dx}\right) = 0\right]dx \rightarrow K\frac{dT}{dx} = C_1$$

$$\int\left[\frac{dT}{dx} = \frac{C_1}{K}\right]dx \rightarrow T(x) = \frac{C_1}{K}x + C_2$$

$$T(x) = \frac{C_1}{K}x + C_2 \rightarrow T(0) = T_L \rightarrow T(0) = \frac{C_1}{K}(0) + C_2 = T_L$$

$$C_2 = T_L$$

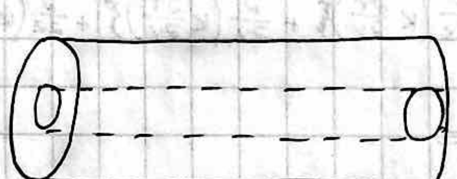
$$T(L) = \frac{C_1}{K}(L) + C_2 = T_R \rightarrow \frac{C_1 L}{K} + T_L = T_R$$

$$C_1 L = (T_R - T_L)K \rightarrow C_1 = \frac{(T_R - T_L)K}{L}$$

$$T(x) = \left(\frac{T_R - T_L}{L}\right)x + T_L \rightarrow q'' = -K\frac{dT}{dx} \rightarrow \frac{dT}{dx} = \frac{T_R - T_L}{L}$$

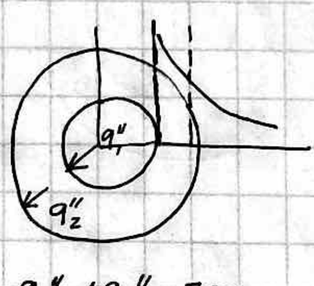
$$\boxed{q'' = -K\left(\frac{T_R - T_L}{L}\right)}$$

EXAMPLE



AXIAL HT: $\rho c \underbrace{\cancel{\frac{\partial T}{\partial t}}}_{\text{S.S.}} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r\cancel{\frac{\partial T}{\partial r}}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\cancel{K\frac{\partial T}{\partial \phi}}\right)^{\text{AXIAL}} + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \cancel{\dot{q}}^{\text{NEG.}}$

$$\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + q$$



$q_1'' \neq q_2''$ BUT

$q_1 = q_2$!

RADIALLY:

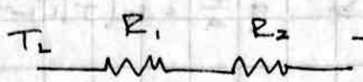
$$0 = \frac{1}{r}\frac{d}{dr}\left(Kr\frac{dT}{dr}\right) \rightarrow$$

$$\frac{d}{dr}\left(Kr\frac{dT}{dr}\right) = 0$$

10/8/2013

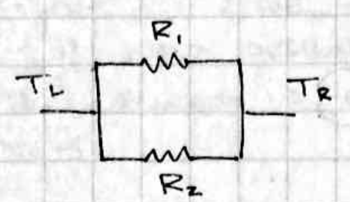
* RESISTANCE ANALOGIES *

$$I = \frac{\Delta V}{R_e}$$



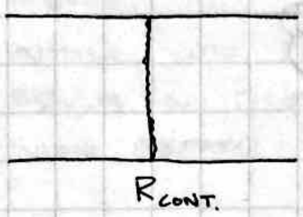
$$R_{TOT} = R_1 + R_2$$

$$\dot{V} = \frac{\Delta P}{R_f}$$



$$\rightarrow \frac{1}{R_{TOT}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$q = \frac{\Delta T}{R_T}$$



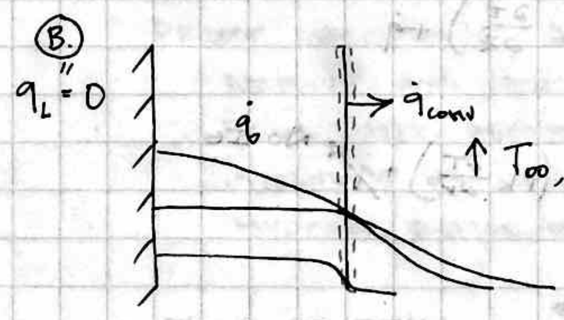
$$q = \frac{KA\Delta T}{L}, \quad R_{COND} = \frac{L}{KA}$$

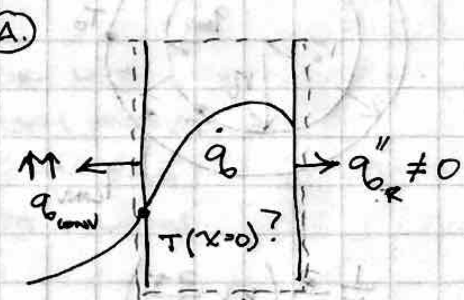
$$q'' = \frac{\Delta T}{R''}$$

10/10/2013

1D, STEADY CONDUCTION w/ GENERATION

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \dot{q}$$





$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) = -\dot{q} \rightarrow \int \frac{d}{dx}\left(K\frac{dT}{dx}\right) = -\dot{q} \, dx \rightarrow \frac{dT}{dx} = \frac{-\dot{q}}{K}x + c_1$$

$$\boxed{T(x) = \frac{-\dot{q}}{2K}x^2 + c_1 x + c_2}$$

$$-K\frac{dT}{dx}\Big|_{x=L} = h\left(T(L) - T_\infty\right)$$

$$\frac{dT}{dx}\Big|_{x=L} = \frac{h}{K}\left[T_\infty - T(L)\right]$$

$$\frac{dT}{dx}\Big|_{x=0} = 0$$

CAN'T SOLVE FOR → NEXT PAGE

10/10/2013

GLOBAL ENERGY BALANCE $\rightarrow$ $\cancel{\frac{dE}{dt}}^{0} = \cancel{\dot{E}_{IN}}^{0} - \dot{E}_{OUT} + \dot{E}_{GEN}$ $\rightarrow$ $\dot{E}_{GEN} = \dot{E}_{OUT}$

$\dot{q}\forall = hA(T(L) - T_\infty)$ $\rightarrow$ $\dot{q}\frac{\forall}{A} = h(T(L) - T_\infty)$ $\rightarrow$ $\dot{q}L = h(T(L) - T_\infty)$

B.C: @ $x = 0$ INTO I

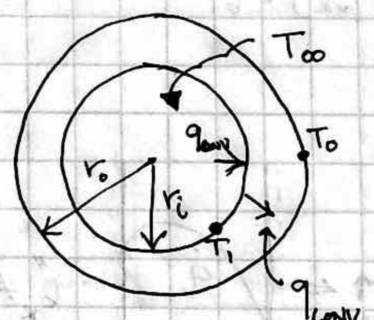
$0 = \frac{\dot{q}}{k}(0) + C_1$, $C_1 = 0$

B.C: @ $x = L$ INTO II'

$T(L) = -\dot{q}/2k\,L^2 + C_2$ $\rightarrow$ $C_2 = T(L) + \dot{q}/2k\,L^2$

$$\boxed{T(x) = \dot{q}/2k\left[L^2 - x^2\right] + T(L)}$$

HW #2 PROBLEM #1



$$\rho c\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}$$

$$0 = \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \cancel{\dot{q}}$$ NO IG

$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)$

$T(r = r_i) =$

$T(r = r_o) = T_o$

NEED TO SOLVE FOR $T_i \rightarrow 371$

$q_{conv} = -k\,2\pi r L\frac{dT}{dr}$

$\hookrightarrow q_r = \frac{-2\pi Lk(T_o - T_i)}{\ln(r_o/r_i)}$ .. $\rightarrow$ RESISTANCE ANALOGY $\rightarrow \frac{T_\infty - T_o}{\frac{1}{hA} + \frac{\ln(R_o/r_i)}{2\pi Lk}}$

* CAN'T USE GLOBAL ENERGY BALANCE, MUST USE INTERFACIAL ENERGY BALANCE

MEETING W/ DR. LIBURDY:

- AJ SENIOR DESIGN TEAM
  - ↳ PROPULSION VS. ENERGY HARVESTING
  - ↳

DESCRIBE RESEARCH CONCEPT, 4 PAGE SHEET ON PROPULSION
→ BE ON A TEAM TO HELP DESIGN/BUILD (MODIFYING) FOR EHARVESTING. DEVICE FOR BACKING UP CFD RESULTS. EMPHASIZE WORKING ON A TEAM.

SYNERGY:

PUTTING INTO PRACTICE WHAT HASS BEEN DONE IN CLASS RESULTS → THEORY. LONG RANGE GOALS OF R.E. HOW WOULD DOING RESEARCH HELP ME.

*SEND:
- RESUME (TO WRITE A LETTER)
- TRANSCRIPT
- WRITTEN SECTIONS OF URISC APP

} BY NEXT MONDAY/WED.

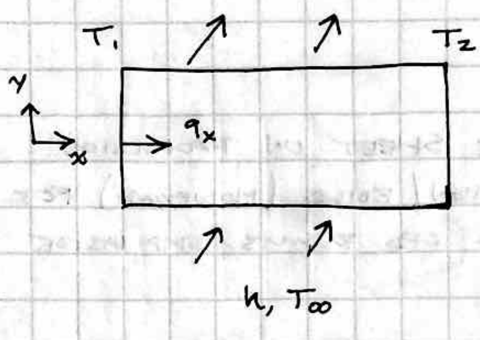
PROPOSAL:

PROJECT DESCRIPTION:
- DEVELOPE AND OPTIMIZE A DEVICE FOR HARVESTING ENERGY FROM WATER. ~~PURPOSE~~ PROJECT IS TEAM FOCUSED, WITH INTENTIONS OF CONSTRUCTING AND OPTIMIZING AN OCILATING HYDROFOIL DEVICE FOR EH AND PROPULSION.

10/15/2013

EXTENDED SURFACE PROBLEMS

-FINS

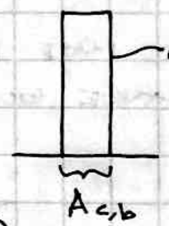


$$q = hA_s (T_s - T_\infty)$$

TO INCREASE q, INCREASE ΔT, h, A_s

→ INCREASIN A_s IS EASIEST SOLUTION

$$\varepsilon = \frac{q_{w/\,fin}}{q_{w/o\,fin}} = \frac{q_{w/\,fin}}{hA_{c,b}\,\theta_b}$$



$$= \frac{\sqrt{hPKA_c}\,\cancel{\theta_b}}{hA_{c,b}\,\cancel{\theta_b}} = \frac{\sqrt{hPKA_c}}{hA_{c,b}}$$

$$\varepsilon_{,\infty\,fin} = \sqrt{\frac{PK}{hA_c}} \rightarrow h\uparrow,\ \varepsilon\downarrow$$

↳ WHY????

FIN EFFICIENCY:

$$\eta = \frac{q_f}{\underbrace{q_{f,MAX}}}$$

HIGHEST WHEN THE ENTIRE LENGTH OF THE FIN IS THE TEMP OF THE BASE

$$\eta_{\infty,fin} = \frac{\sqrt{hPKA_c}\,\cancel{\theta_b}}{hA_f\,\cancel{\theta_b}}$$

$$q_f = \eta\, q_{f,MAX}\ ,\quad q_f = \eta\, hA_f\,\theta_b$$

→ RESISTANCE ANALOGIES ←

$$R_f = \frac{\theta_b}{q_f} = \frac{1}{\eta h A_f}$$

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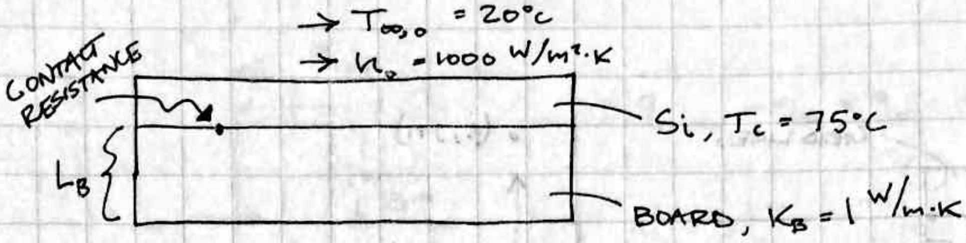
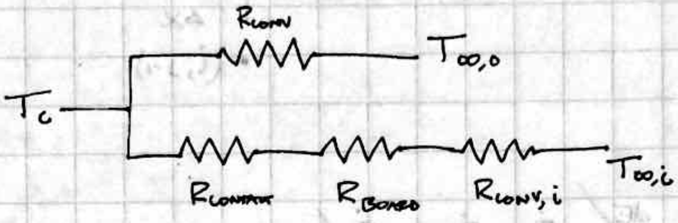


$L_B = 5$ mm



$$q_{chip,\ no\ fins} = q_{top,\ no\ fins} + q_{bottom}$$

$$\frac{R_{contact}}{A_c} \qquad \frac{L_B}{K_B A_c}$$

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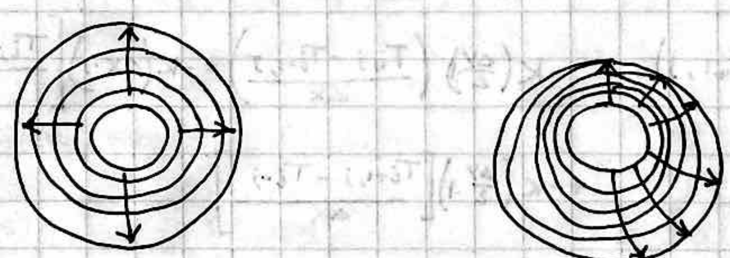
$$q_f = \eta_f q_{f,max} \rightarrow q_f = \eta_f A_f h \theta_b$$

CH. 4 2-D SS CONDUCTION:



1.) GRAPHICAL

2.) ANALYTICAL

3.) APPROXIMATE

   A.) SHAPE FACTORS

   b.) NUMERICAL

   - FINITE DIFFERENCE
   - FINITE VOLUME

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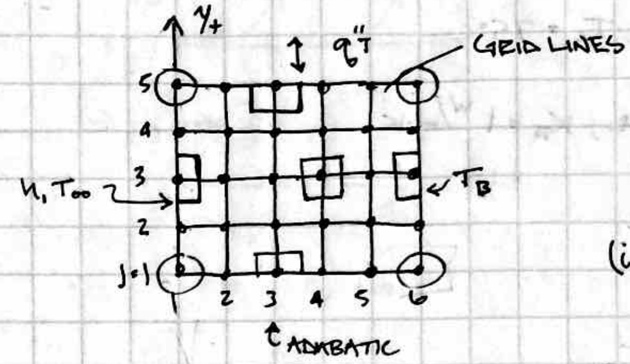
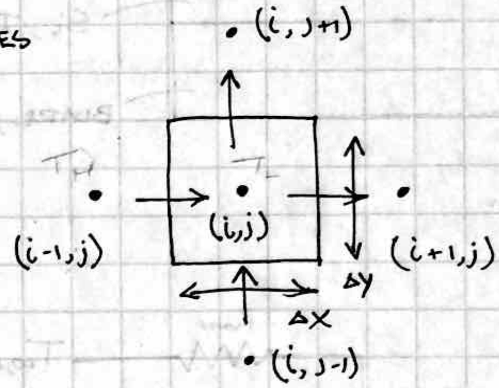
→ EXAMPLE ←





ENERGY BALANCE

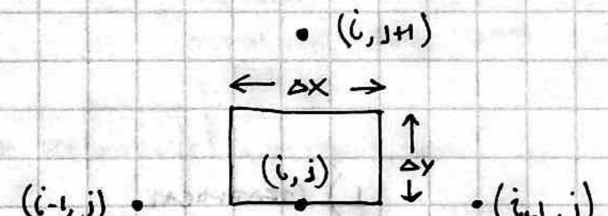
$$q'' \begin{cases} +x \\ +y \end{cases} \rightarrow \cancel{\frac{dE}{dt}} = \dot{E}_{in} - \dot{E}_{out} + \cancel{\dot{E}_{gen}}$$

$$q_L + q_B = q_R + q_T \rightarrow \text{CONDUCTION} \quad q = -KA\frac{dT}{dx}$$

$$-K(\Delta y \cdot 1)\left[\frac{T_{i,j} - T_{i-1,j}}{\Delta x}\right] - k(\Delta x \cdot 1)\left[\frac{T_{i,j} - T_{i,j-1}}{\Delta y}\right] + k(\Delta y \cdot 1)\left[\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\right]$$

$$+k(\Delta x \cdot 1)\left[\frac{T_{i,j+1} - T_{i,j}}{\Delta y}\right] = 0 \rightarrow T_{i,j} = \frac{1}{4}\left[T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}\right]$$

→ BOTTOM BOUNDARY ←



$$q_L = q_T + q_R$$

$$-K\left(\frac{\Delta y}{2}\cdot 1\right)\left(\frac{T_{i,j} - T_{i-1,J}}{\Delta x}\right) = -k(\Delta x \cdot 1)\left[\frac{T_{i,j+1} - T_{i,j}}{\Delta y}\right]$$

$$+ -k\left(\frac{\Delta y}{2}\cdot 1\right)\left[\frac{T_{i+1,j} - T_{i-j}}{\Delta x}\right]$$

$$\left(\frac{1}{2} + 1 + \frac{1}{2}\right)T_{i,j} = \frac{1}{2}T_{i+1,j} + \frac{1}{2}T_{i-1,j} + T_{i,j+1}$$

$$T_{i,j} = \frac{1}{2}\left[\frac{1}{2}T_{i+1,j} + \frac{1}{2}T_{i-1,j} + T_{i,j+1}\right]$$

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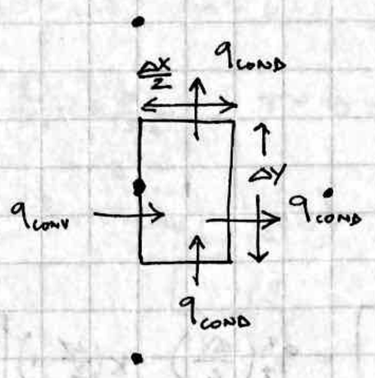
LEFT BOUNDARY

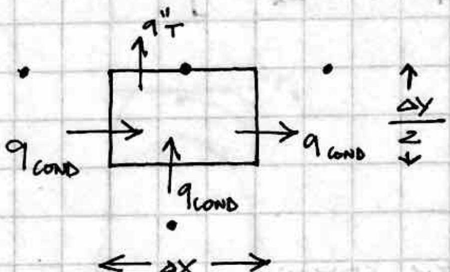


$$q_{conv} + q_B = q_R + q_T$$

$$h(\Delta y \cdot 1)\left[T_\infty - T_{i,j}\right] - k\left(\frac{\Delta x}{2}\cdot 1\right)\left[\frac{T_{i,j} - T_{i,j-1}}{\Delta y}\right]$$

$$= k(\Delta y \cdot 1)\left[\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\right] - k\left(\frac{\Delta x}{2}\cdot 1\right)\left[\frac{T_{i,j+1} - T_{i,j}}{\Delta y}\right]$$

TOP BOUNDARY



$$q_L + q_B = q_R + q''_T(\Delta x \cdot 1)$$

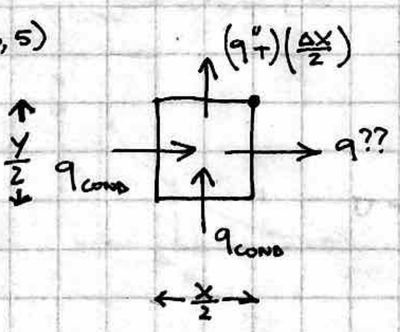
$$-k\left(\frac{\Delta y}{2}\cdot 1\right)\left[\frac{T_{i,j} - T_{i-1,j}}{\Delta x}\right] + -k(\Delta x \cdot 1)\left[\frac{T_{i,j} - T_{i,j-1}}{\Delta y}\right] = -k\left(\frac{\Delta y}{2}\cdot 1\right)\left[\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\right] + q''_T(\Delta x \cdot 1)$$

RIGHT BOUNDARY:

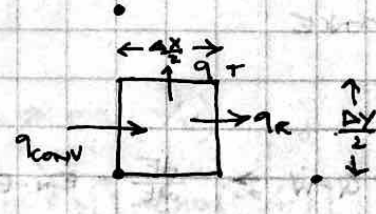
$$T_{i,j} = T_B, \quad i = 6, \quad 2 \le j \le 4$$

CORNER NODES:

(6, 5)



(1,1)



$$h\left(\frac{\Delta y}{2}\cdot 1\right)\left(T_\infty - T_{1,1}\right) = -k\frac{\Delta x}{2}\left[\frac{T_{i,j+1} - T_{i,j}}{\Delta y}\right]$$

$$-k\frac{\Delta y}{2}\left[\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\right]$$

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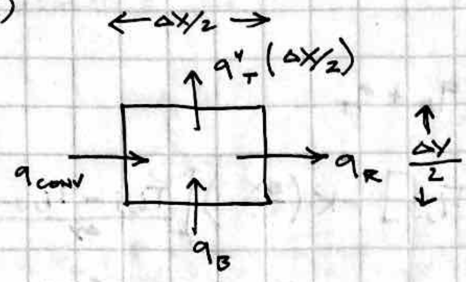
(1,5)



$$h\left(\frac{\Delta y}{2}\cdot 1\right)\left[T_\infty - T_{i,j}\right] + -k\left(\frac{\Delta x}{2}\cdot 1\right)\left[\frac{T_{i,j} - T_{i,j-1}}{\Delta y}\right] = q''_T\left(\frac{\Delta x}{2}\right) - k\left(\frac{\Delta y}{2}\cdot 1\right)\left[\frac{T_{i+1,j} - T_{i,j}}{\Delta x}\right]$$

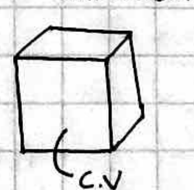
$$q_{LHS} = \sum_{j=1}^{5} A\left[T_\infty - T_{i,j}\right] \qquad q_{RHS} = -\sum_{j=1}^{5} AK\left[\frac{T_{6,j} - T_{5,j}}{\Delta x}\right]$$

$\hookrightarrow (\Delta y \cdot 1)$ 'OR' $\left(\frac{\Delta y}{2}\cdot 1\right)$

$q''_T A_T + q_{RHS} + q_{LHS} = 0 \rightarrow$ USE TO CHECK SPACING

10/22/2013 $\rightarrow$ TRANSIENT CONDUCTION

WITHOUT SPACIAL VARIATION



W/ 1-D SPACIAL VARIATION

$\rightarrow$ HDE $\leftarrow$

APPROXIMATE
* (1 TERM ∞ SERIES)
* NUMERICAL

EXACT (∞ SERIES)

LUMPED CAPACITANCE

$T = f(t)$

COE $\quad \frac{dE}{dt} = \dot{Q} - \dot{W} \rightarrow \frac{dE}{dt} = \cancel{\dot{E}_{IN}} - \dot{E}_{OUT} + \dot{E}_{GEN} \rightarrow E = U$

$U = mu \rightarrow \frac{d}{dt}(mu) = m\frac{du}{dt} + u\cancel{\frac{dm}{dt}} = -q_{conv}$

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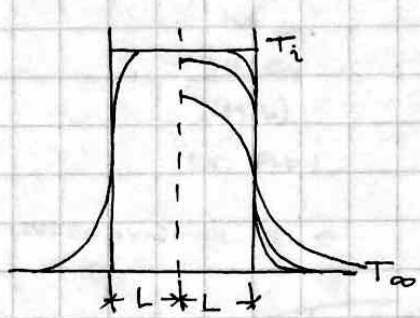
$du = c\,dT$, $m = \rho V$, $\rho V c \frac{dT}{dt} = -hA_s(T - T_\infty)$, $\theta = T - T_\infty$

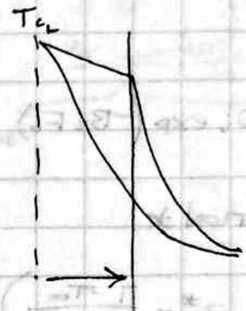
$\frac{d\theta}{dt} = \frac{dT}{dt}$, $T = \theta + T_\infty \rightarrow \frac{dT}{dt} = \frac{d\theta}{dt} + 0$

$\frac{d\theta}{dt} = -hA_s\left(\frac{1}{\rho V c}\theta\right) \rightarrow \frac{d\theta}{\theta} = \frac{-hA_s}{\rho V c}dt$

$\hookrightarrow \int_{\theta_i}^{\theta}$ $\qquad \hookrightarrow \int_0^t$

$\ln\left(\frac{\theta}{\theta_i}\right) = \frac{-hA_s}{\rho V c}(T - 0)] \rightarrow \frac{\theta}{\theta} = \exp\left(\frac{-t}{\tau}\right)$, $\tau = \underbrace{\left(\frac{1}{hA_s}\right)}_{R_{conv}}\underbrace{\rho V c}_{C_t}$

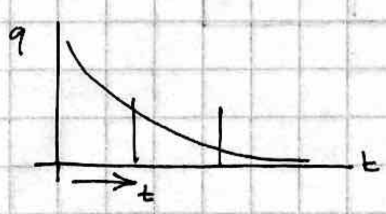




R_{cond}	R_{conv}
$\frac{L}{kA}$	$\frac{1}{hA}$
$\frac{L}{kwH}$	$\frac{1}{hwH}$

$\frac{hL}{k} << 1 \rightarrow \boxed{Bi < 0.1} \leftarrow$ LUMPED BEHAVIOR

$q_{conv} = hA_s\theta \ \{W\}$



$q = Ah\theta_i \exp(-t/\tau)$

$Q = \rho V c\,\theta_i[1 - \exp(-t/\tau)]$

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$$Bi = \frac{hL_c}{K}$$

VOLUME / AREA

	$L_c = \frac{V}{A_s}$	$\frac{1}{2}$ THICKNESS
PLANE	L	L
CYLINDER	$r_o/2$	r_o
SPHERE	$r_o/3$	r_o

← IF THIS IS LUMPED, YOU'RE LUMPED!

* TEST W/ THIS

$$\tau = \left\{\frac{1}{hA_s}\right\}(\rho V c)$$

$$\theta = \theta_i \exp(-t/\tau) = \theta_i \exp(-Bi\, F_o)$$

*TRANSIENT 1-D CONDUCTION *

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K \frac{\partial T}{\partial x}\right) \quad , \quad \theta^* = \frac{T - T_\infty}{T_i - T_\infty}$$

$$\frac{x}{L} = x^*$$

$$F_o = \frac{\alpha t}{L^2}$$

IC $\quad T(x, t=0) = T_i$

2BC $\quad \frac{\partial T}{\partial x}(x=0, t) = 0$

$$\left.\frac{\partial T}{\partial x}\right|_{x=L} = hA\left(T(x=L, t) - T_\infty\right)$$

NON-DIMENTIONALIZED EQN'S, SOMEONE SOLVED ANALYTICALLY

		EXACT	APPROX. $F_o > 0.2$	Q
$Bi = \frac{hL}{K}$	PLANE WALL THICKNESS 2L	$\theta = \sum_{n=1}^{\infty} C_n \ldots$	1 TERM of $C_1 = f(C_1)$	
$Bi = \frac{hr_o}{K}$	CYLINDER	$\theta^* = \sum_{n=1}^{\infty} C_n \ldots$		
	SPHERE			

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1-D TRANSIENT CONDUCTION ← *PRINT EQUATION SHEETS*

- FINITE DIMENTION
- INFINITE MEDIUM
  1.)
  2.)
  3.)

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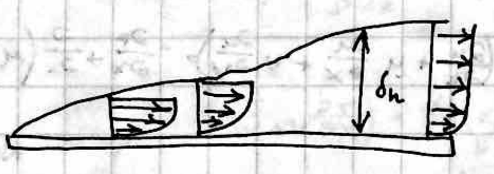
~~* CHAPTER 6 *~~

CONVECTION

- THEORY (CH. 6) EX. FLOW
- APPLICATIONS
  - INTERNAL FLOW (CH. 8)

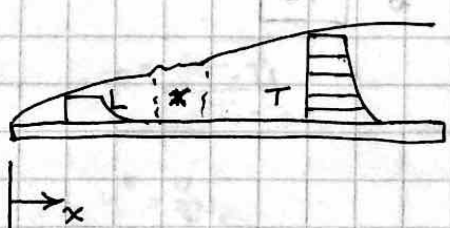
→ HYDRODYNAMIC BL ←

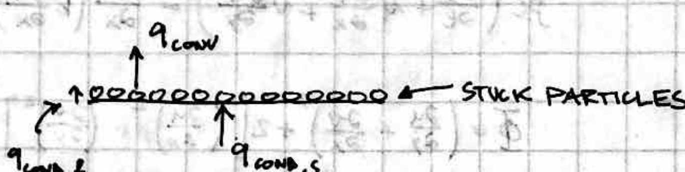


$\tau_s A_s = F_D \rightarrow \tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0}$

$\frac{\partial u}{\partial y} \approx \frac{u_\infty - \cancel{u_0}^{\,0}}{\delta_h} \rightarrow \tau_s = \frac{1}{L}\int \tau_{s,x}\,dx$ $\boxed{C_f = \frac{1}{L}\int C_{f,x}\,dx}$

→ THERMAL BOUNDARY LAYER ←





$q_{conv} = q_{cond,f}$

$hA(T_s - T_\infty) = -k_f A \frac{\partial T_s}{\partial y}\Big|_{y=0}$

$$\boxed{h = \frac{-k_f \frac{\partial T}{\partial y}}{T_s - T_\infty}}$$

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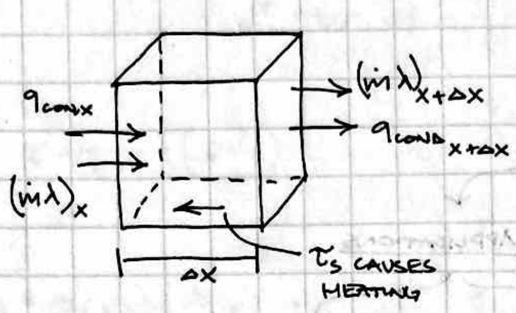
$$q = \bar{h}A(T_s - T_\infty) \rightarrow q \stackrel{?}{=} f(x) \rightarrow \checkmark$$

$$\bar{h} = \frac{1}{L}\int h_x \, dx \leftarrow \text{AVERAGE}$$

* ENERGY EQUATION: DIFFERENTIAL



3 EQUATIONS: → 2-D, → INCOMPRESSIBLE

$$\underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{\text{INCOMP.}} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \leftarrow \text{CONTINUITY}$$

N-S:

$$X: \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\underbrace{\left(\mu\frac{\partial u}{\partial y}\right)}_{\text{NEWTONIAN}}$$

$$Y: \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)$$

ENERGY:

VISCOUS DISSAPATION

$$\rho c\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} + \boxed{\mu\Phi}$$

$$\Phi = \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]$$

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* 2-D BOUNDARY LAYER FLOW:

- ASSUMPTIONS:

* $u >> v$

* $\frac{\partial}{\partial x} << \frac{\partial}{\partial y}$

* $\frac{\partial P}{\partial x} \approx \frac{dP_\infty}{dx}$

* S.S

* $\rho, \mu, k \Rightarrow$ CONST.

$$\rho\left(\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{S.S} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\underbrace{\cancel{\frac{\partial P}{\partial x}}}_{\frac{-dP_\infty}{dx}} + \frac{\partial}{\partial x}\left(\mu \underbrace{\cancel{\frac{\partial u}{\partial x}}}_{NEG.}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{-dP_\infty}{\rho\, dx} + \nu\frac{\partial^2 u}{\partial y^2}}$$

$$\rho c\left(\underbrace{\cancel{\frac{\partial T}{\partial t}}}_{S.S.} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(\underbrace{\cancel{k\frac{\partial T}{\partial x}}}_{NEG.}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \cancel{\dot{q}} + \mu\Phi$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c}\left(\frac{\partial u}{\partial y}\right)^2$$

→ NONDIMENTIONALIZING THESE EQUATIONS ←

$$T^* = \frac{T - T_s}{T_\infty - T_s}$$

$$u^* = \frac{u}{V}, \quad v^* = \frac{v}{V} \quad \text{FREE STREAM}$$

$$x^* = \frac{x}{L}$$

$$y^* = \frac{y}{L}$$

$$\boxed{P_R = \frac{\nu}{\alpha} \qquad Re_L = \frac{\rho V L}{\mu}}$$

$P_r \approx 1.0$ GASES

$P_r << 1$ liquid METALS

$P_r >> 1$ OILS

$$\boxed{P_r \propto \frac{\delta}{\delta t}}$$

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$\tau_s = \mu \frac{\partial u}{\partial y}$ SUBSTITUTE N.D.T ... $\frac{\mu V}{L} \frac{\partial u^*}{\partial y^*}$

$$C_f = \frac{\tau_s}{\frac{1}{2} \rho V^2} = \frac{2}{Re_L} \frac{\partial u^*}{\partial y^*}$$

$$h = \frac{k_f}{L} \frac{\partial T^*}{\partial y^*} \rightarrow \frac{\partial T^*}{\partial y^*} = \frac{hL}{k_f} \rightarrow \boxed{Nu = \frac{hL}{k_f}}$$

PROBLEMS TO STUDY FOR MIDTERM

- ECCENTRIC WIRE (EX. IN CLASS)

*REVIEW SESSION

→ MAKE SURE YOU CAN WORK EX. PROBLEMS → SWITCH UP KNOWNS / UNKNOWNS

HW#4 P2 ~~t~~ Q USE LUMP
P3 t Q USE APROX.

4 PROBLEMS ON EXAM

→ CH. 3 THEORETICAL → SURFACE / GLOBAL ENERGY BALANCE
RADIATION B.C. / CONDUCTION (KNOWN ΔT)

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \cancel{\dot{q}} \rightarrow \text{INT. TWICE, APPLY B.C.}$$

NEG.

↳ DO PROBLEMS IN RAD. / CART. W/ GEN.

→ CH. 4 2-D

NUMERICAL

ENERGY BALANCE APPROACH (q^+, x^+, y^+)

WILL HAVE TO HAVE EXPLICIT EQ. IE. $T(i,j) = \ldots$ (KNOWNS)

MUST WRITE EQ.'S IN TERMS OF WHATS KNOWN

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KNOW HOW TO FIND PERIM. FOR OTHER THAN PIN FIN.

FIN EFFECTIVNESS → IS IT WORTHWHILE TO PUT FINS ON?
- KNOW OTHER TYPES OF FINS

CH. 5 TRANSIENT

<u>LUMPED</u>
$T(t)$

<u>SPACIAL-TEMPORAL</u>
APPROXIMATE
$F_o > 0.2 \quad T(t,x)$

$$Q = \int_0^t q_b \, dt$$

→ "USED 'LUMPED CAPACITANCE', PROVE WHY ANALYSIS IS GOOD"

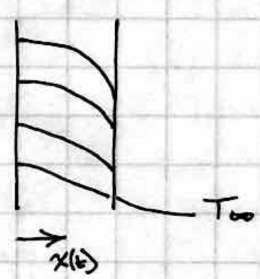
EX. 5.5, 5.6

BE ABLE TO DRAW ALL TEMPERATURE PROFILES

→ SOLVING FOR TIME w/ APPROXIMATION.

$T(x,t) \rightarrow t_? \; T(L,t) = 6 \; ?$

$t_? \; T(0,t) = 6 \; ?$



$$\theta^* = \theta_o^* \cos(\zeta_1 x^*) \, C_1 \exp(-\zeta^2 F_o)$$

$\theta^* \rightarrow \dfrac{T - T_\infty}{T_i - T_\infty}$; $x^* \rightarrow \dfrac{x}{L}$; F_o: TIME EMBEDED ← ASSUMES FINITE OBJECT

MIDTERM REVIEW:

CHAP 3: 1-D, S.S. CONDUCTION
CHAP 4: 2-D, S.S. CONDUCTION
CHAP 5: TRANSIENT CONDUCTION

EX.
3.8, IN-CLASS FIN PROB. w/ RECT. FINS (3.146)

HW #3 PROB. 2

HW #5: EX. 5.5, 5.6

BIOT NUMBER: RATIO OF HEAT TRANSFER RESISTANCES INSIDE & AT THE SURFACE OF A BODY.

FOURIER NUMBER: RATIO OF DIFFUSIVE / CONDUCTIVE TRANSPORT RATES

FIN PROBLEM
NUMERICAL PROBLEM W/ RAD B.C.
TRANSIENT CONDUCTION PROBLEM
REVIEW HW PROBLEMS
TEMP. DISTRIBUTIONS

* EXTENDED SURFACES *

FIN PERFORMANCE:

FIN EFFECTIVENESS: THE RATIO OF THE FIN HEAT TRANSFER RATE TO THE HEAT TRANSFER THAT WOULD EXIST W/ OUT THE FIN

$$\boxed{\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b}} \longleftarrow > 2 \text{ FOR}$$

WHERE $A_{c,b}$ IS THE FIN X-SECTIONAL AREA

$$\boxed{\varepsilon_f = \left(\frac{kP}{hA_c}\right)^{1/2}} \quad > 4 \text{ FOR}$$

$$\boxed{\varepsilon_f = \frac{R_{t,b}}{R_{t,f}}}$$

FIN EFFICIENCY: RATIO OF FIN HEAT TRANSFER (ACTUAL) TO MAXIMUM POSSIBLE HEAT TRANSFER (PURE CONVECTIVE)

$$\boxed{\eta_f = \frac{q_F}{hA_f\theta_b}}$$

10/31/2013

NOTES: CHAP. 7 → EXTERNAL FLOW

EXTERNAL FLOW → CONTINUITY, X-MOM, Y-MOM, ENERGY

BLASIUS:

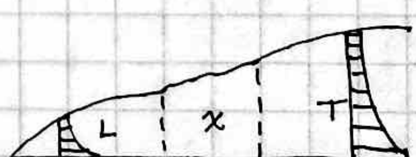
LAM. FLOW OVER A FLAT PLATE

ANALYTICAL

1 ANALYTICAL SOLUTION

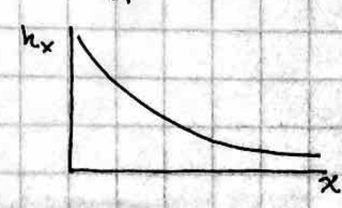
$$\left.\begin{array}{l}\text{LAMINAR} \\ T_s = \text{CONST.} \\ \text{FLAT PLATE}\end{array}\right\} \text{CONVECTION} \rightarrow q = hA(T_s - T_\infty)$$

$Re_x \leq 5\times10^5$



$$\left.\frac{\partial T}{\partial y}\right|_{x_1} > \left.\frac{\partial T}{\partial y}\right|_{x_2}$$

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$$



$$\boxed{\bar{h}_x = \frac{1}{x}\int_0^x h_x\,dx}$$

10/31/2013

$$C_{f,x} = \frac{\tau_s A}{\frac{1}{2}\rho V_\infty^2 A} \begin{matrix} \leftarrow \text{DRAG FORCE} \\ \hline \leftarrow \text{INERTIAL FORCE} \end{matrix}$$

$$Pr = \frac{\nu}{\alpha} \equiv \frac{\text{MOMENTUM DIFF}}{\text{THERMAL DIFF.}} \quad , \quad Nu_x = \frac{\text{CONVECTION}}{\text{CONDUCTION}}$$

$$\boxed{\begin{aligned} \overline{Nu_x} &= \frac{h_x x}{k} \\ \overline{C_{f,x}} &= \frac{1}{x}\int_0^x C_{f,x}\,dx \end{aligned}}$$

LAMINAR S.S. FLOW * ONLY ANALYTICAL SOLUTION IN CHAPTER *

$$C_{f,x} = 0.664 Re_x^{-1/2}$$

$$Nu_x = 0.332 Re^{1/2} Pr^{1/3}$$

LAMINAR: 1/2

TURBULENT: Nu (exp + 1)

C_f (exp)

$$\frac{1}{x}\int_0^x h_x\,dx \rightarrow h_x = -k\frac{\partial T}{\partial x}\left(\frac{1}{T_s}\right.$$

$$\rightarrow \frac{hx}{k} = 0.332 Re^{1/2} Pr^{1/3}$$

$$h = \frac{1}{x} 0.332 k Re_x^{1/2} Pr^{1/3}$$

$$h = 0.332 k Re^{-3/2} Pr^{1/3}$$

$$\int h = 0.332 k Re_x^{-1/2} Pr^{1/3}$$

$$\boxed{\overline{Nu_x} = 0.664 k Re^{1/2} Pr^{1/3}}$$

T_s = CONST.

FLAT PLATE

S.S. FLOW

- LAM
  - LOCAL
  - AVE
- MIXED (L+T)
- TURBULENT

10/31/2013

FOR FLAT PLATE

$\left.\begin{array}{l} T_s = \text{CONST} \\ \text{S.S} \\ \text{TURBULENT} \end{array}\right\} Re_{cr} < Re_x < 10^8$

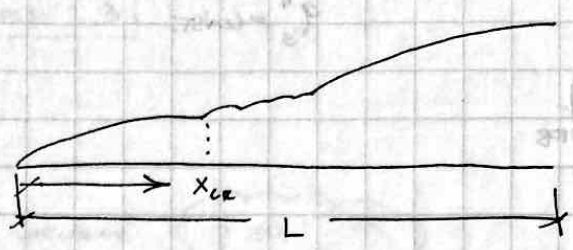
LOCAL: $C_{f,x} = Re_x^{-1/5}$

$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$

$\delta_h = 0.37 x Re_x^{-1/5}$, $\delta_t \approx \delta_h$

MIXED B.L. $\overline{Nu_x}$

$\overline{Nu_x} = \frac{\overline{h_x} x}{k}$

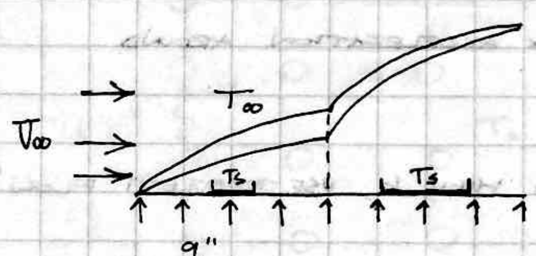


$\overline{h_x} = \frac{1}{x}\left[\int_0^{x_c} h_{x\,lam} dx + \int_{x_{cr}}^{x} h_{x,TURB} dx\right]$

$\overline{Nu_x} = (0.037 Re_x^{4/5} - A) Pr^{1/3}$

$\overline{Cf_x} = 0.074 Re^{-1/5} - \frac{2A}{\sqrt{Re_x}}$

CONSTANT HEAT FLUX



BOTH VARY w/ x

$q'' = h_x(T_{s,x} - T_\infty) \rightarrow \frac{q''}{h_x} + T_\infty = T_{s,x}$

$q_L'' = \overline{h_x}\left(\overline{T_{s,x} - T_\infty}\right)$

$Nu_x = 0.453 Re_x^{1/2} Pr^{1/3}$ $(Pr \geq 0.6)$ LAM!

$Nu_x = 0.0308 Re_x^{4/5} \cdot Pr^{1/3}$ $(0.6 < Pr < 60)$ TURB!

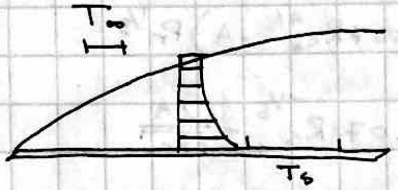
$\left(\overline{T_s - T_\infty}\right) = \frac{q'' L}{k \overline{Nu_L}} \rightarrow lam \rightarrow \left(\frac{1}{x}\right)\left(\int_0^L \frac{A}{k/x} dx\right) dx \frac{k}{x}$

11/5/2013

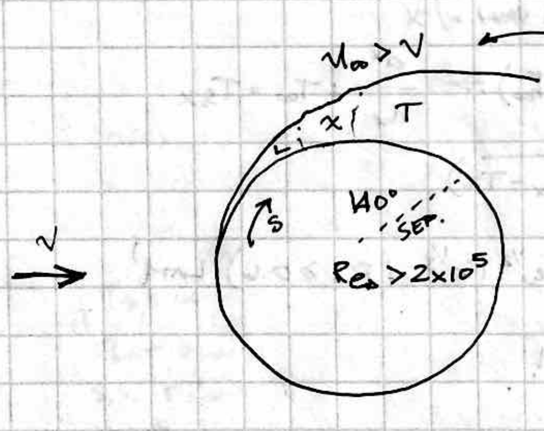
EXTERNAL FLOW

- FLAT PLATE
  - T_s = CONST.
    - LAM
    - MIXED
    - TURB
  - q_s'' = CONST.
- BLUNT BODIES
  - SPHERES (IN TEXT)
  - TUBES
    - SINGLE TUBES
    - TUBE BANKS

$Re_{crit} \approx 2\times10^5$

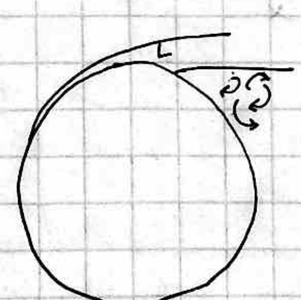


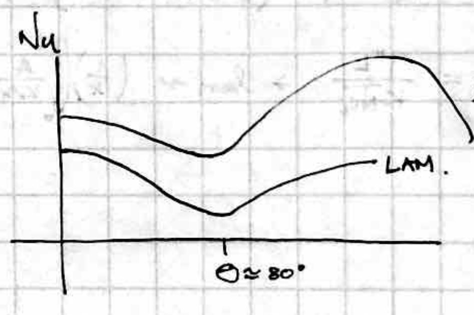
$$h = \frac{k\frac{\partial T}{\partial x}}{T_s - T_\infty} \text{ AS } x \uparrow$$



← THERE IS FLOW ACCELERATION AROUND THE SPHERE

"WE WANT A HIGH h, USE TURBULENT FLOW"





X LOOK @ GRAPH IN BOOK

* FIG. 7.10 *

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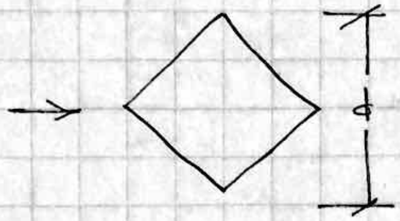
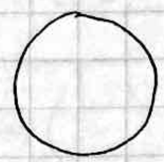
OTHER SHAPES:



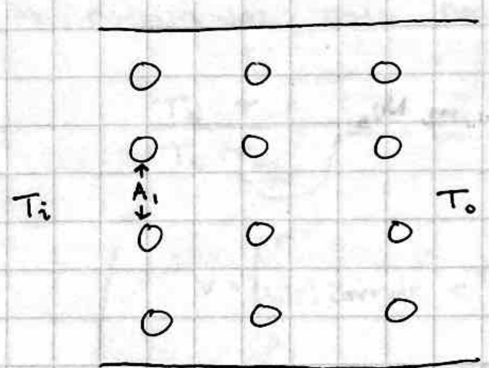
$$\overline{Nu}_D = C Re_D^m Pr^{1/3}$$

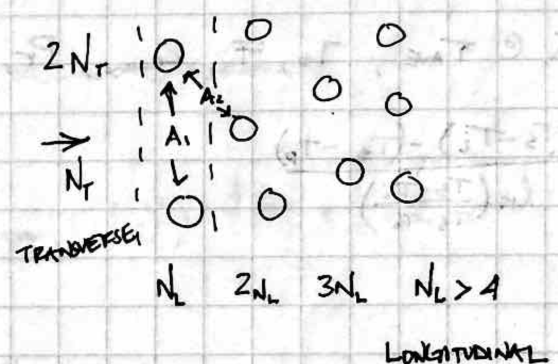
$\boxed{\text{TABLE 7.3}}$



$\boxed{\text{TABLE 7.2}}$ CIRCULAR CYLINDER (EQN. 7.54)

TUBE BANKS





$$\overline{Nu}_D = C_1 C_2 Re_{D,max} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/2}$$

EVALUATE PROPERTIES @ T_i, T_o → $\overline{T} = \frac{T_i + T_o}{2}$

Pr_s = PRANDTL # EVAL. @ SURFACE TEMP.

$$q = N \pi D L \bar{h} \left(T_s - \overline{T}_m\right)$$

$\left(T_s - \overline{T}_m\right)$ → ΔT_{LM} → log MEAN TEMPERATURE DIFFERENCE

$$\overline{Nu}_D = \frac{\bar{h} D}{k}$$

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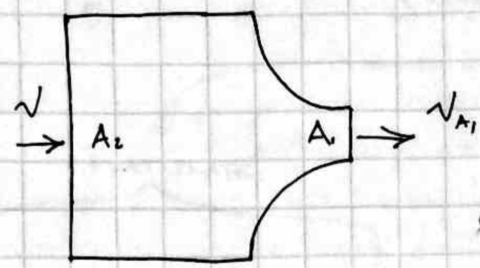

$\rho A_2 V = \rho A_1 V_1$

$S_T V = (S_T - D) V_{A1}$

$V_{A1} = \left(\frac{S_T V}{S_T - D}\right)$

IF $S_D < \frac{S_T + D}{2}$

FIND T_o → 7.63 DERIVE IN HW

$0 = \dot{Q} - \cancel{\dot{W}}^{N.W} + \dot{m}(h_i) - \dot{m}_e h_e$

$\dot{Q} \rightarrow q = N\pi DL(T_S - \bar{T}_m)$

FIND ΔT LOG-MEAN

- FIND $\bar{h}$ @ T_{AVE}, T_o, $\bar{T}$, Re_{max}, Pr, C_1, m, Nu_D

$$\Delta T = \frac{(T_S - T_i) - (T_S - T_o)}{\ln\left(\frac{T_S - T_i}{T_S - T_o}\right)}$$

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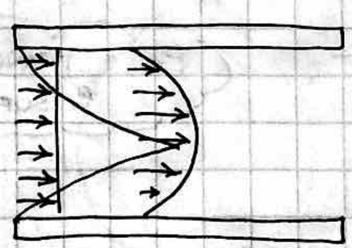
INTERNAL FLOW:



THERMAL

(L): $Le_t = 0.05 Re_D D Pr$

(T): $Le_t = 10D$

FULLY DEVELOPED WHEN: $\frac{\partial u}{\partial x} = 0$

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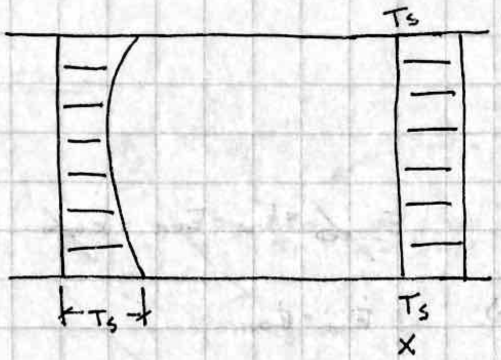
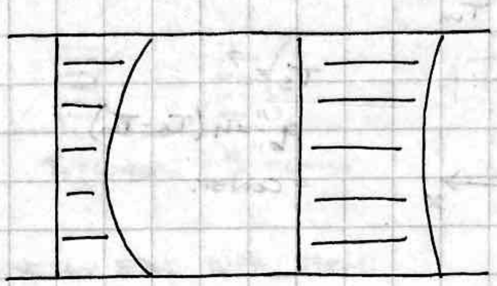


CONSTANT SURFACE TEMPERATURE

$$q_f'' = q_s'' = -K_f \left.\frac{\partial T}{\partial r}\right|_{r=R}$$



CONSTANT HEAT FLUX

NON-DIMENTIONAL TEMP. PROFILE

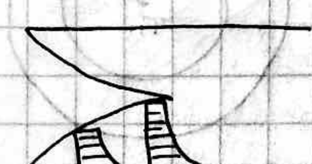
$$\frac{T_s - T}{T_s - T_m}$$ → BULK FLUID TEMPERATURE

$$V = \int_0^r u(r)\, 2\pi r\, dr = u_{max}\left(1 - \left(\frac{r}{R}\right)^2\right)$$ → TEMPERATURE: $\dot{E}_x = \dot{m} c T = \rho A u c T$ ↳ $u(t)$

→ DERIVATION IN BOOK (BULK FLUID TEMPERATURE)

$T_m = T_m(x)$, $T(r, x)$, $T(s) = T_s(x)$ IF q''_{const}

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0$$ → FULLY DEVELOPED FOR $x > l_{e,t}$

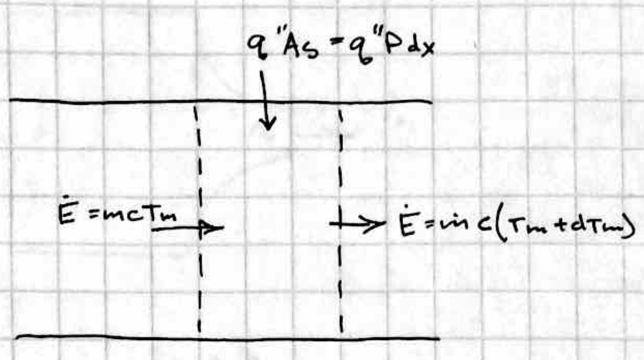


→ IN FULLY DEVELOPED REGION:

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0$$

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right) = \frac{-\partial T/\partial r}{T_s - T_m} = \frac{h}{K}$$ — $h(x) \neq f(x)$ IN F.D. REGION

GLOBAL ENERGY BALANCE

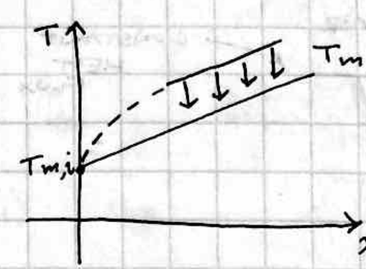


$$\cancel{\dot E_{stored}} = \dot E_{in} - \dot E_{out} + \cancel{\dot E_{gen}}$$

$$\dot E_{in} = \dot E_{out}$$

$$\boxed{q'' P dx = \dot m c\, dT_m}$$

$$\boxed{\frac{dT_m}{dx} = \frac{q_s'' P}{\dot m c}}$$



$T_s = ?$

$q_s'' = h(T_s - T_m)$

$= \text{CONST.}$

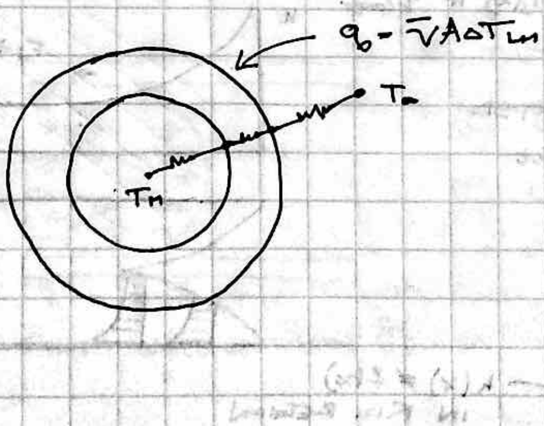
T_s = CONST.

$$\left.\begin{aligned} &q'' P dx = \dot m c\, dT \\ &\hookrightarrow h(T_s - T_m) \end{aligned}\right\} T_s - T_m = \Delta T \rightarrow \frac{d\Delta T}{dx} = \frac{dT_s}{dx} - \frac{dT_m}{dx} = \frac{-dT_m}{dx}$$

$$h \Delta T P dx = \dot m c\, d\Delta T \rightarrow \int_{\Delta T_i}^{\Delta T_x} \frac{d\Delta T}{\Delta T} = \int_0^x \frac{-hP}{\dot m c} dx \rightarrow \ln\left(\frac{\Delta T_x}{\Delta T_i}\right) = -\left(\frac{hP}{\dot m c}\right) x$$

$$\boxed{\frac{T_s - T_{m,x}}{T_s - T_{m,i}} = \exp\left(\frac{-\bar h P x}{\dot m c}\right)}$$

$\hookrightarrow \rho V A$



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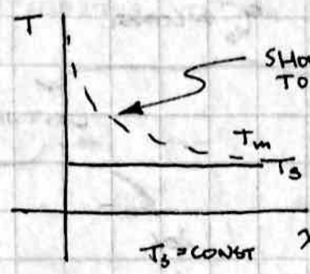
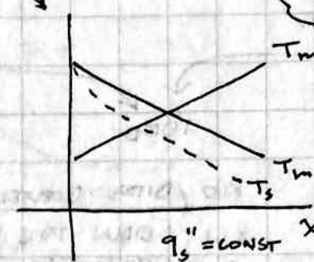
RECALL:

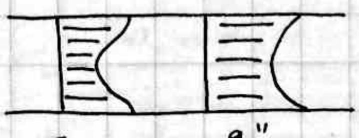
BE ABLE TO DRAW ALL PROFILES / DISTRIBUTIONS FOR HEATING / COOLING

DIST = $f(x)$





$q_s'' = \dot{m}c(T_o - T_i)$



$$T_m = \frac{1}{A_c}\int T u\, 2\pi r\, dr \quad , \; T = f(r, u)$$

→ ENERGY EQN. DIFF FORM:

$$u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \rightarrow T(x,r) \rightarrow T_m = \frac{1}{u_{max}A_c}\int_0^{r_o} T u r\, 2\pi r\, dr$$

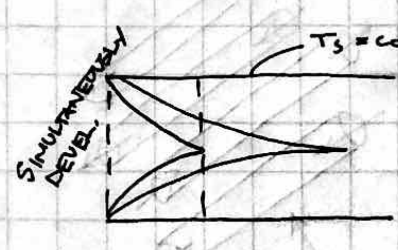
$\boxed{Nu_D = (\text{CONST } q_s'') = 4.36}$ $\boxed{T_s = \text{CONST}, \; Nu_D = 3.66}$

LAMINAR FLOW

ENTRANCE LENGTHS:

$$\left.\begin{matrix} L_{e_h} = 0.05 Re_D D \\ L_{e_T} = 0.05 Re_D Pr \end{matrix}\right\} \text{LAM} \qquad \left.\begin{matrix} L_{e,h} = 10D \\ L_{e,t} = 10D \end{matrix}\right\} \text{TURB}$$

T_s = CONST.



$$q_o = \bar{h} A \Delta T_{LM} \rightarrow Nu_D = f(Re_D, Pr) \left.\begin{matrix} \rightarrow \\ \rightarrow g,H \end{matrix}\right\} \overline{T_m}$$

FINDING $\bar{h}$:

CIRCULAR TUBES

T_s = CONST

q_s'' = CONST

LAM → $x > L_{e,t}$, $x < L_{e,h}$

TURB

LAM

CIRCULAR TUBES

- T_s = CONST.
  - LAMINAR
    - F.D.R
    - DEVELOPING
      - $x > L_{e,h}$, $x < L_{e,t}$ (T.D) (8.58)
      - $x < L_{e,h}$, $x > L_{e,t}$ (S.D) (8.58)
  - TURB
- q_s'' = CONST.
  - TURB
    - 8.60 DITTUS - BOELTER
    - 8.61 SEIDER - TATE
    - 8.62 GNIELINSKI
  - LAMINAR
    - dev → BOOK
    - F.d

S.D. = SIMULTANEOUS DEVELOPEMENT

EXAM OUTLINE:

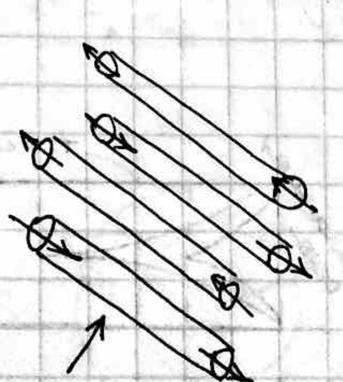
- SPHERE (EXT)
  - HEAT / COOL
  - LAM / TURB
- CYLINDERS (EXT)
  - SINGLE → T_s OR T_∞ CONST / OR q_s'' = CONST.
  - TUBE BANK → T_s

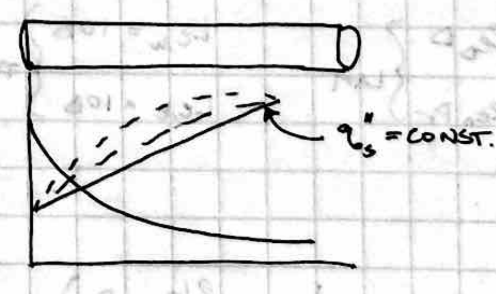
$\boxed{\text{TEMP. DISTRIBUTIONS / PHYSICS BASED QUESTIONS}}$

INTERNAL FLOW → LAM (T_s = CONST.), TURB (q_s OR T_s = CONST)

CHAPTERS 6, 7, 8 & POSSIBLY 11

<u>CH. 11 NOTES:</u>





$T_{m,o} - T_{m,i}$

$\ln($

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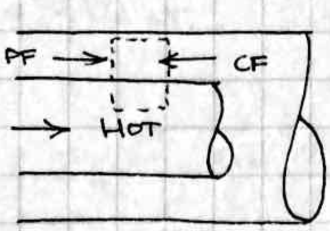
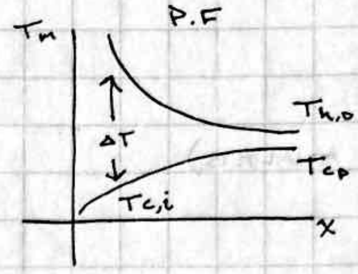
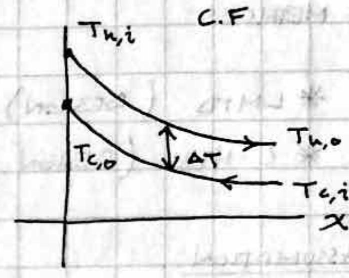
* HEAT EXCHANGERS *

— CONCENTRIC TUBES







$$\Delta T_m = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

PARALLEL:

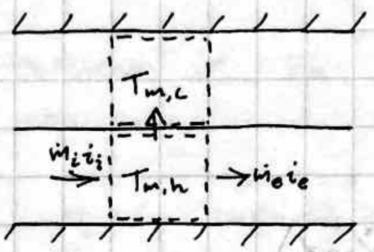
$\Delta T_1 = T_{h,i} - T_{c,i}$

$\Delta T_2 = T_{h,o} - T_{c,o}$

COUNTER:

$\Delta T_1 = T_{h,i} - T_{c,o}$

$\Delta T_2 = T_{h,o} - T_{c,i}$



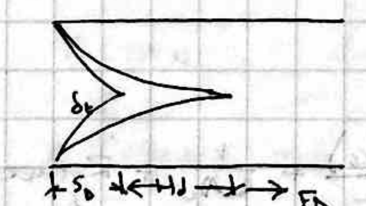
$q = \bar{U}A(T_{m,h} - T_{m,c})$

SAME

$\dot{m} i_i = \dot{m} i_e + q$

$$Nu_D = \left[\int Nu_{D_{SD}} dx + \int \cancel{Nu_{D_{ED}} dx} + \int Nu_{D_{FD}} dx\right]\frac{1}{L} \rightarrow Nu_D = \left(\frac{L_{e,t}}{L}\right)\overline{Nu}_{D_{SD}}$$

NO CORRELATION



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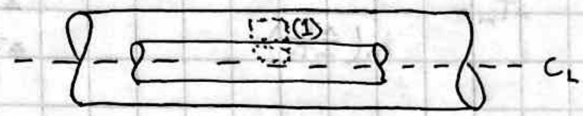
CH. 11 DESIGNING / ANALYZING HEAT EXCHANGERS
↳ "SELECTING"

2 METHODS:

* LMTD (DESIGN)
* ε-NTU (DESIGN & ANALYSIS)

<u>ASSUMPTION:</u>

- INSULATED
- S.S.
- ΔPE & ΔKE NEG.
- CONST. THERMAL PHYS. PROP.
- NEG. AXIAL CONDUCTION



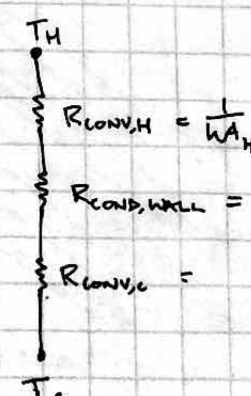
C.V. #1

$$0 = q - \dot{W} + \dot{m}_i\left(i_i + \cancel{\frac{V_i^2}{2}} + \cancel{gz_i}\right) - \dot{m}_o\left(i_o + \cancel{\frac{V_o^2}{2}} + \cancel{gz_o}\right)$$

INCOMPRESSIBLE FLUID → $di \approx c\,dT$

$(+)\ q = \dot{m}_H c_H (T_{H,i} - T_{H,o})$ → (HOT STREAM)

$(+)\ q = \dot{m}_c c_c (T_{c,o} - T_{c,i})$



$$R_{TOT} = \frac{1}{(hA)_H} + \frac{\ln(r_o/r_i)}{2\pi KL} + \frac{1}{(hA)_c} + R_{OTHER}$$

CONTACT RES.
FOULING RES.

$\bar{U}A = U_iA_i = U_oA_o$

$$\boxed{q = \bar{U}A\Delta T_{LM}}$$

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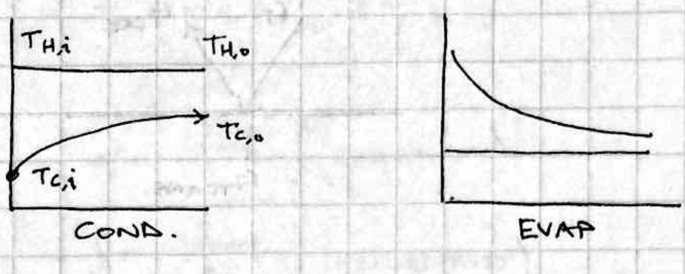
$q = \dot{m}_c c_c (T_{c,o} - T_{c,i})$

$C_c, C_H \rightarrow$ HEAT CAPACITY

$q = \dot{m}_H C_H (T_{H,i} - T_{H,o})$

$q = \bar{U} A \Delta T_{LM}$



LOG-MEAN TEMP. DIFFERENCE METHOD

- EASIEST IF $T_{H,i}$, $T_{c,i}$, $\dot{m}_H$, $\dot{m}_c$ & EITHER $T_{c,o}$ OR $T_{H,o}$ SPECIFIED

$q = C_c (T_{c,o} - T_{c,i})$, $q = C_H (T_{H,i} - T_{H,o})$

$q = \bar{U} A \Delta T_{LM}$

$\rightarrow h_o A_o$

$\rightarrow h_i A_i$

ε - NTU

EASIEST IF A_o OR A_i

ε = EFFECTIVENESS = $\frac{q}{q_{MAX}}$ $\longrightarrow$ USE TO FIND $q = \varepsilon q_{MAX}$

NTU = NUMBER OF TRANSFER UNITS $= \frac{\bar{U}A}{C_{MIN}}$ $\rightarrow$ $C_r = \frac{C_{MIN}}{C_{MAX}}$

$\frac{1}{\bar{U}A} = R_{CONV,i} + R_{COND} + R_{CONV,o}$

$q = q_{MAX} \varepsilon$

$q = C_c (T_{c,o} - T_{c,i})$

$q = C_H (T_{H,i} - T_{H,o})$

CALC NTU $= \frac{\bar{U}A}{C_{MIN}}$

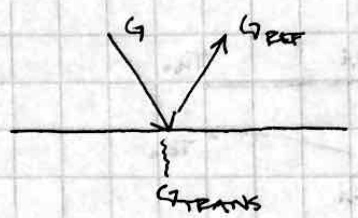
USE TABLE 11.3 $\rightarrow \varepsilon = f(NTU, C_r)$

11/26/2013

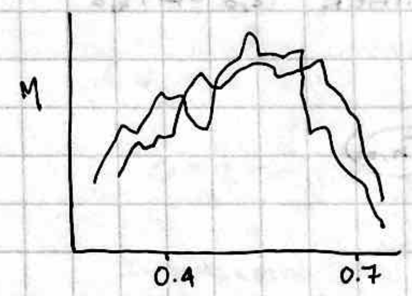
* RADIATION: CHAP. 12, 13 FUND. OF RADIATION

- EVERYTHING EMITS RADIATION, EVERYTHING RECIEVES

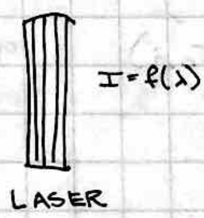
$$\boxed{G = G_{ABS} + G_{REF} + G_{TRANS}}$$

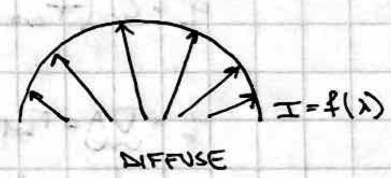


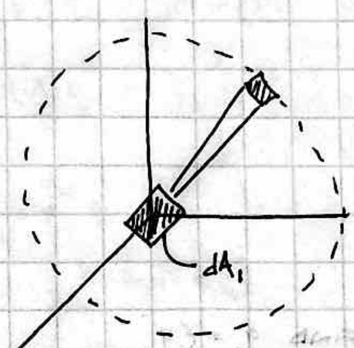
ELECTROMAGNETIC WAVES:



DIRECTIONALITY:







$$E_\lambda = \int I \, dA_s \rightarrow E = \int E_\lambda \, d\lambda$$

$E = f(T)$ $G \neq f(T)$

EMISSION $E = \varepsilon \sigma T^4$

$E_b = \sigma T^4$ (BLACKBODY EMISSION)

BLACKBODY:
HIGHEST AMOUT, PERFECT ABSORBER

RADIOSITY: J = NET STUFF LEAVING

$$\boxed{J = E + G_{REF}}$$

$$\boxed{\alpha = \frac{G_{ABS}}{G}} \quad \text{(ASORBTIVITY)}$$

$$\boxed{\rho = \frac{G_{REF}}{G}} \quad \text{(REFLECTIVITY)}$$

$$\boxed{\tau = \frac{G_{TRAN}}{G}} \quad \text{(TRANSMISSTIVITY)}$$

$$\boxed{J = E + \rho G}$$

NET RADIATIVE FLUX FROM SURFACE

$$q'' = J - G = E + \rho G - G \rightarrow q'' = E - \alpha G$$

$\alpha \sigma T_{sur}^4$

$$\boxed{q'' = F_{12} \varepsilon \sigma \left(T_{sur,1}^4 - T_{sur,2}^4\right)}$$

12/3/2013

REFRESHER:

- RADIOSITY $J = E + \rho \underbrace{G}_{G_{REF}}$
- IRRADIATION $G = G_{ABS} + G_{REF} + \underbrace{G_{TRANS}}_{=0 \text{ IF OPAQUE}}$
- EMISSION $E = \varepsilon \sigma T^4$

DIFFUSE $E, J, G \neq f(\phi, \theta)$

DIFFUSE SURFACE

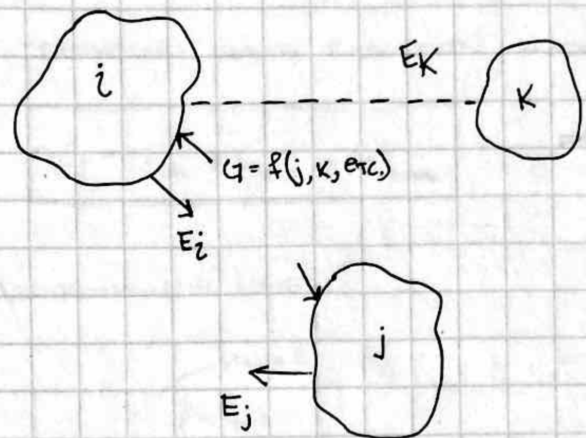
$\alpha \cong \varepsilon$

GRAY $\varepsilon < 1$

OPAQUE $(\tau = 0)$

$\boxed{\varepsilon_{BB} = 1}$ $\boxed{\alpha + \rho + \tau = 1}$

12/3/2013



$q_{i \to j} = A_i J_i F_{ij}$

↳ VIEW FACTOR LOOKING FROM $i \to j$

$q_{j \to i} = A_j J_j F_{ji}$

$$\boxed{q_{ij} = q_{i \to j} - q_{j \to i}}$$

→ BLACKBODY RADIATION, HOW MUCH IS REFLECTED?

$J = E + \rho G \rightarrow \epsilon E_b + (1 - \alpha) G$

$J = \varepsilon E_b + (1 - \varepsilon) G \quad J = E_b$

$$\boxed{q_{ij} = A_i J_i F_{ij} - A_j J_j F_{ji}}$$

RULES FOR FINDING F_{ij} F_{ji}

1.) $A_i F_{ij} = A_{ji}$ REPROCITY

2.) $1 = \sum_{j=1}^{N} F_{ij}$

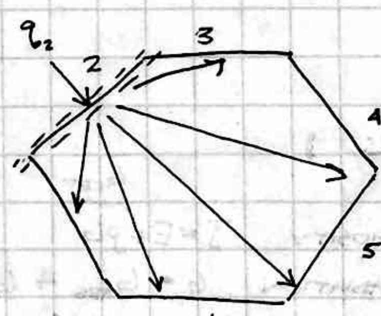
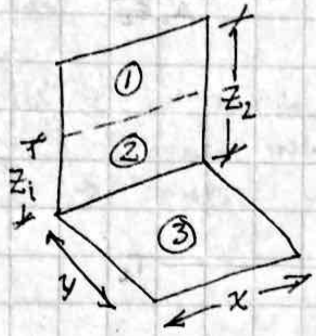


TABLE 13.2, PG. 867 → PERP. RECT.



F_{23} = ☺

F_{21} = ?

$F_{2(1,3)}$ = ☺

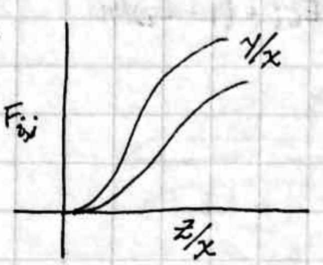
F_{23} : y/x, z/x $\quad F_{23} = 0.125$

$$F_{2(2,1)}: \left.\begin{aligned} y/x &= 0.6 \\ z/x &= 0.4 \end{aligned}\right\} \begin{aligned} F_{2(3,2)} &= 0.19 \\ F_{21} &= 0.065 \end{aligned}$$

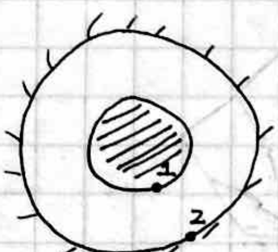
$$A_1 F_{12} = F_{21} A_2$$

$$\boxed{F_{12} = \frac{A_2}{A_1} F_{21}} \leftarrow R$$

13.6



EX.



$X = \frac{N(N-1)}{2}$

N → 2 SURFACES → 4F → N^2

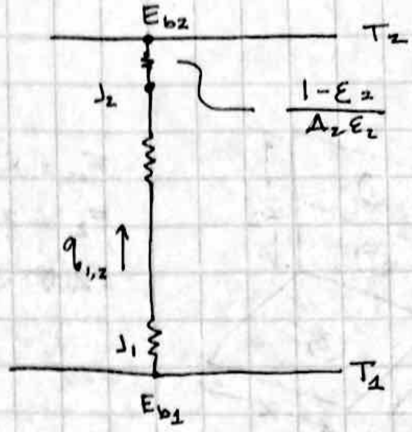
F_{12}

F_{21} FIND ALL BUT X #

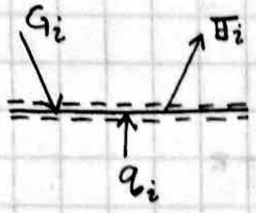
F_{22} OF F FROM TWO RULES

F_{11}

$$q_{ij} = A_i F_{ij} (J_i - J_j) = \frac{J_i - J_j}{\left(\frac{1}{A_i F_{ij}}\right)} = R$$



SURFACE RESISTANCE



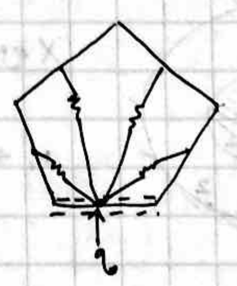
$$q_i = A_i (J_i - G_i)$$

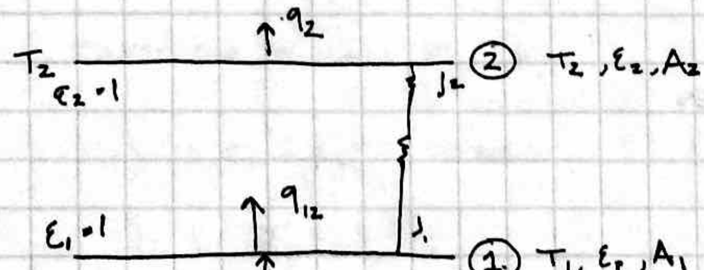
$$J_i = \varepsilon_i E_{bi} + (1 - \alpha_i) G_i \quad , \quad J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i$$

$$G_i = \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}$$

$$q_i = A_i \left(J_i - \frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i} \right) = A_i \varepsilon_i \frac{(E_{bi} - J_i)}{1 - \varepsilon_i}$$

$$\boxed{q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{(1/A_i F_{ij})}}$$

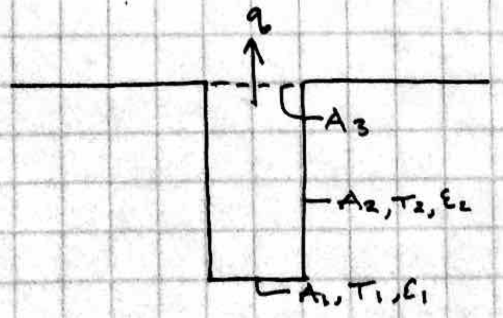




$$q_{12} = \frac{E_{b1} - E_{b2}}{\sum R} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1 - \varepsilon_1}{A_1 \varepsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \varepsilon_2}{A_2 \varepsilon_2}}$$

ALL IMAGINARY SURFACES ARE TREATED AS A BLACKBODY

MAKING ENCLOSURE

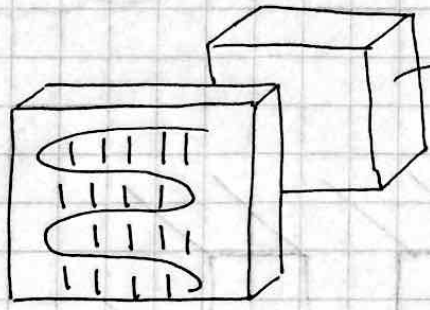


12/5/2013

* FINAL EXAM

HX → EXTERNAL FIN
→ CONV INSIDE
→ CONVEC OUTSIDE

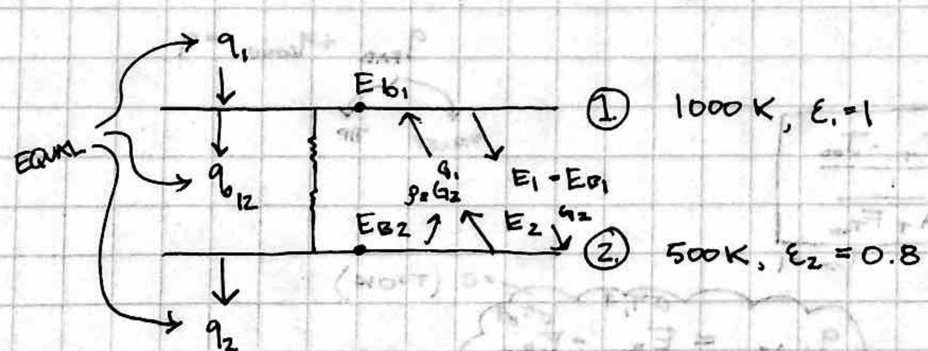
→ EX. RADIATOR IN A CAR:



HOT ENGINE BLOCK, RADIATION TO FINS

RADIATION:

$$q_{b_1} = \frac{E_{b_1} - J}{\frac{1 - \varepsilon_1}{A_1 \varepsilon_1}} \iff q_{12} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}}$$



A.) G_1 (WHATS COMING IN)
B.) J_1 (WHATS GOING OUT)
C.) J_2
D.) q''

$G_1 = \rho_2 G_2 + E_2$

$G_1 = \rho_2 E_{b_1} + E_2 = (1-\alpha)\sigma T_1^4 + \varepsilon_2 \sigma T_2^4$

↓ GRAY

a.) $\boxed{G_1 = (1 - \varepsilon_2)\sigma T_1^4 + \varepsilon_2 \sigma T_2^4}$

12/5/2013

$$J = E + \rho G \xrightarrow{(1-\alpha) = (1-\varepsilon) = 0} J_1 = E_{B_1} = \sigma T_1^4 \quad \text{b.}$$

$$J_2 = E_2 + \rho_2 G_2$$

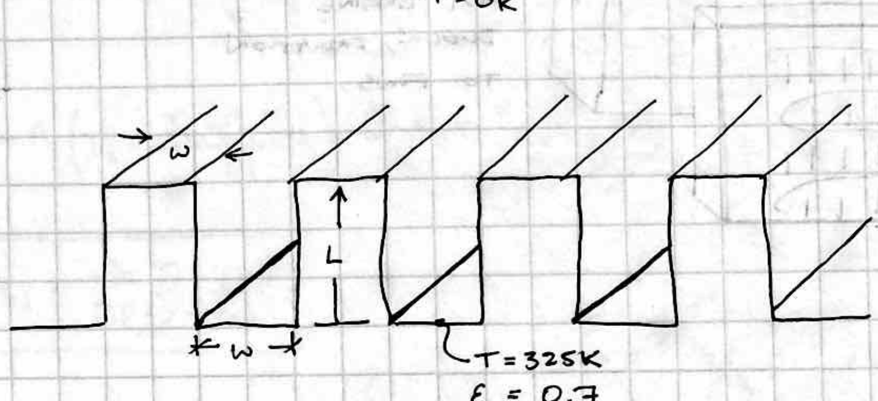
$$J_2 = \varepsilon_2 \sigma T_2^4 + (1-\varepsilon_2)\sigma T_1^4 \quad \text{c}$$

$$q_1'' = J_1 - G_1 = J_1 - J_2$$

$$q_1'' = \sigma T_1^4 - J_2 \quad \text{c}$$

KNOW NON-CIRCULAR DUCTS FOR FINAL

13.51



$L = 0.125\,m$

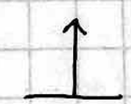
$W = 0.025\,m$ → ASSUME ~~SO~~ THIN ATMOSPHERE, SO

$$q = q_{TIP} + q_{VALLEY}$$

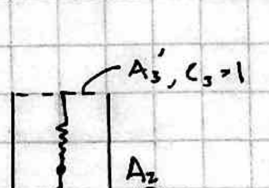
→ TIP ←

$q_{RAD} + q_{CONV} = q$

VALLEY TIP



$$\boxed{q_T = \frac{J_T - J_\infty}{\frac{1}{A_T \underbrace{F_{T\infty}}_{=1}}}}$$



$A_1' = W$ (PER LENGTH)

$$q_{(12)3} = \frac{\overbrace{E_{B_{12}}}^{\sigma T_1^4} - \overbrace{E_{B_3}}^{\sigma T_3^4 = 0\ (T = 0K)}}{\frac{1-\varepsilon_1}{A_{12}\varepsilon_1} + \frac{1}{A_{12} F_{(12)3}}}$$

$$A_{12}' = F_{(12)(3)} = A_3' F_{3(12)} = A_3'$$