10/1 DYNAMIC SYSTEMS AND CONTROLS

HOMEWORK NOT GRADED

QUIZES WEEKLY → QUESTIONS LIKELY OFF HOMEWORK

LIKELY GOING TO USE SIMULINK

PROJECT PROPOSAL DUE OCT. 11TH
- WHO ARE YOU WORKING WITH
- WHAT ARE YOU GOING TO DO (1 PAGE) "SEE SOMETHING FROM END TO END"

NOTES:

CONTROL SYSTEMS: CONTROL OUTPUT OF PROCESS | PLANT

PLANT INPUT → $\boxed{\text{PROCESS PLANT}}$ → OUTPUT

PLANTS: CAR, PLANE, FURNACE, ROBOT ARM

INPUTS: THROTTLE, LEVER ON THERMOSTAT, INPUT CURRENT

4 COMPONENTS:

PLANT
ACTUATORS
SENSORS
COMPENSATOR

THROTTLE → $\boxed{\text{CAR}}$ → SPEED

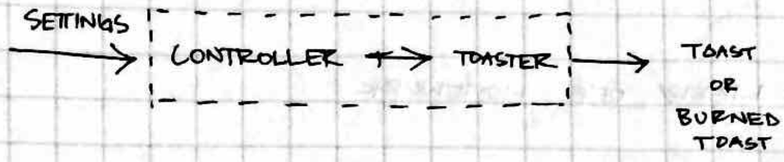
VALVE POSITION → $\boxed{\text{ROOM}}$ → TEMP

TYPES OF CONTROL

* OPEN LOOP: NO FEEDBACK

INPUT → CONTROLLER → PLANT → OUTPUT

10/1/2013



HOW WELL BREAD IS TOASTED ISN'T FEEDBACK INTO SETTINGS

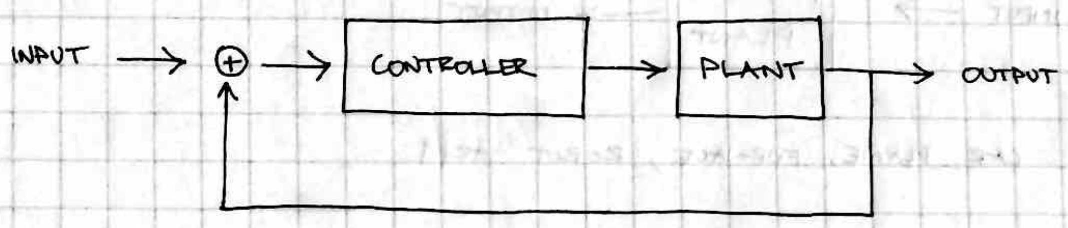
POSITIVES:

- SIMPLE, NO SENSING REQUIRED

ISSUES: /

ASSUMES GOOD

* CLOSED LOOP CONTROL → USES DYNAMIC SENSOR INFO: COMPARE ACTUAL OUTPUT W/ DESIRED OUTPUT



INPUT - OUTPUT = Δ

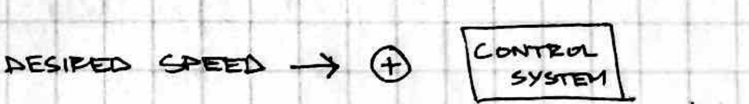
ADVANTAGES:

- ACCURACY
- REDUCE IMPACT OF DISTRBANCES
- FASTER RESPONSE

ISSUES:

- COSTLY
- POTENTIALLY UNSTABLE

PHYSICAL REPRESENTATION



10/1/2013

LAPLACE TRANSFORM REVIEW

→ USED IN SOLVING DIFF. EQ'S

$$\frac{d^3 f(t)}{dt^3} + \frac{6d^2 f(t)}{dt^2} + \frac{13 d f(t)}{dt} + 2 f(t) + 10 = y(t) + \frac{d^4 y(t)}{dt^4}$$

DEFINITION: $\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t) e^{-st} dt$

$s = \sigma + j\omega$

- $F(s)$ EXISTS IF
  1.) $f(t)$ IS SECTIONALLY CONTINUOS (FINITE # OF DISCONTINUITIES)
  2.) EXPONENTIAL ORDER

$$\int |f(t)| e^{-st} < \infty$$ ← DOES NOT GROW FASTER THAN EXPONENTIAL

$$|f(t)| < Me^{at},\ a < \infty.\quad @\ t = \infty,\ e^{-st} \to 0$$

EXAMPLE:

$$f(t) = t^2 u(t) \to \mathcal{L}[t^2 u(t)] = \int_0^\infty t^2 e^{-st} dt \to \text{IBP} \to$$

$u = t^2 \qquad dv = e^{-st}$

$du = 2t \qquad v = -\frac{1}{s} e^{-st}$

$$\to t^2\left(-\frac{1}{s}\right)e^{-st}\Big|_0^\infty - \int_0^\infty 2t\left(-\frac{1}{s}\right)e^{-st} dt$$

$$t^2 e^{-st}\Big|_0^\infty \quad \underbrace{\frac{t^2}{e^t} = \frac{2t}{e^t} = \frac{2}{e^t}}_{\text{L'HOPITAL}} \to 0 \qquad 0e^0 = 0$$

$$\to \text{IBP AGAIN} \to 0 - 0 + \frac{2}{s}\left[\int t e^{-st} dt\right]$$

$u = t \qquad du = e^{-st}$

$du = dt \qquad v = -\frac{1}{s} e^{-st} dt$

$$= \frac{2}{s}\left[t\left(-\frac{1}{s}\right)e^{-st}\Big|_0^\infty - \int -\frac{1}{s} e^{-st} dt\right. \to \ldots$$

WILL BE PROVIDED A LAPLACE TRANSFORM TABLE

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OFFICE HOURS ⇒ M: 5-6 PM, W: 2-3 PM

THANE: T: 2PM - 3PM
SEPILE: R: 10 AM - 11 AM

LAPLACE TRANSFORM SHEET ON BB

+ WORKSHEET

* REQUIRMENT FOR LAPLACE TRANSFORM:
- EXPONENTIAL ORDER

$$e^{-at}|f(t)| < M \quad \text{FOR ALL } t > t_0$$

1.) $f(t) = e^{bt}\sin ct \rightarrow |f(t)| = |\cancel{e^{bt}}\sin ct| \leq \cancel{e^{bt}}$

$\hookrightarrow |\sin ct| \leq 1 \quad a = b,\ M = 1$

2.) $f(t) = t^n \quad \lim_{t \to \infty}\left[e^{-at}t^n\right] = 0$ ← IMPLYES EXPONENTIAL ORDER

3.) $f(t) = e^{t^2} \rightarrow e^{-at}|f(t)| = e^{-ab}e^{t^2} \rightarrow = e^{t^2 - at} \rightarrow$

w/ t @ ∞, $e^{t^2-at} = \infty$

* MUST BE SECTIONALLY CONTINUOUS *

NOTES:

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}dt, \quad s = \sigma + j\omega$$

→ PROPERTIES ←

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = \int_0^\infty \frac{df}{dt}e^{-st}dt \rightarrow \quad u = e^{-st} \quad du = -se^{-st} \quad dv = \frac{df}{dt}dt \quad v = f$$

$$= f(t)e^{-st}\Big|_0^\infty - (-s)\int_0^\infty f(t)e^{-st}dt = s\mathcal{L}[f(t)] + \underbrace{f(t)e^{-\infty}}_{=0 \text{ (EX. ORDER)}} - f(0)$$

$$= sF(s) - f(0)$$

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→ IN GENERAL ←

$$\mathcal{L}\left(\frac{df}{dt}\right) = sF(s) - f(0)$$

$$\mathcal{L}\left(\frac{d^2f}{dt^2}\right) = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\left(\frac{d^nf}{dt^n}\right) = s^nF(s) - s^{n-1}f(0) \ldots - f^{n-1}(0)$$

DERIVATIVES

$$\mathcal{L}\left(\int_0^{\tau} f(t)dt\right) = \frac{1}{s}F(s)$$

INTEGRATION

$$\mathcal{L}(f(t-a)) = e^{-as}F(s)$$

TIME SHIFT

$$\mathcal{L}[e^{-at}f(t)] = F(s+a)$$

FREQ. SHIFT

$$\mathcal{L}[f_1(t) + \ldots + f_n(t)] = \mathcal{L}[f_1(t)] + \mathcal{L}[f_n(t)] = F_1(s) + \ldots F_N(s)$$ (LINEARITY)

$$\mathcal{L}[af(t)] = aF(s)$$

INVERSE LAPLACE TRANSFORM

DON'T WORRY ABOUT THIS.

$$\mathcal{L}[F(s)] = f(t) = \frac{1}{2\pi j}\int_{\sigma+j\omega}^{\sigma-j\omega} F(s)e^{st}dt$$

EX.

$$F_1(s) = \frac{1}{(s+4)^2} \rightarrow L^{-1}\left[\frac{1}{s^2}\right] = tu(t) \rightarrow s = s+4 \text{ (FREQ. SHIFT)}$$

$$\rightarrow e^{-4t} \rightarrow \boxed{f_1(t) = e^{-4t}\,t\,u(t)} \rightarrow \text{P.F.E} \rightarrow F_1(s) = \frac{1}{(s+4)^2} = \frac{A}{(s+4)} + \frac{B}{(s+4)^2}$$

$$1 = A(s+4) \ldots$$

* INITIAL VALUE THEORUM *

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

* FINAL VALUE THEORUM *

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

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1.) ALL ROOTS OF DENOM. OF $F(s)$ HAVE NEGATIVE REAL PARTS, OTHERWISE $F(s)$ OSCILLATES (NOT STABLE)

2.) $F(s)$ HAS NO MORE THAN ONE REAL POLE @ ORIGIN, OTHERWISE $\rightarrow 1/0 \rightarrow \infty$

EX.

$$F(s) = \frac{4}{s(s+4)}$$ (1.) (2.) → BOTH SATISFIED

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{4}{s+4} = 1$$

EX.

$$F(s) = \frac{4}{s(s-4)} \rightarrow sF(s) \rightarrow \frac{4}{s-4}$$ ... DOES NOT SATISFY 1.)

EX. $$\underbrace{\frac{s^3+2s^2+6s+7}{s^2+s+7}}_{\text{HOW DO WE DO THIS}}$$

* ROAD MAP:

1.) CONVERT FROM TIME TO FREQ. USING L.T
2.) SOLVE NEW EQUATIONS
3.) GET INTO FORM THAT IS RECOGNIZED
4.) CONVERT BACK TO TIME DOMAIN

* PARTIAL FRACTION EXPANSION:

$$F(s) = \frac{2}{(s+1)(s+2)} = \frac{K_1}{(s+1)} + \frac{K_2}{(s+2)}$$ * RESIDUAL METHOD

→ MULT. BY $s+1$ → $\frac{2}{s+2} = K_1 + \frac{(s+1)K_2}{s+2}$ → MAKE $s=-1$ (SEND $K_2 \to 0$)

$$\frac{2}{-1+2} = K_1 \rightarrow \{K_1 = 2\} \rightarrow \{K_2 = -2\}$$

$$F(s) = \frac{2}{s+1} - \frac{2}{s+2} \rightarrow \boxed{(2e^{-t} - 2e^{-2t})u(t)}$$

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*** ALGEBRAIC METHOD ***

$$\frac{2}{(s+1)(s+2)} = \frac{K_1}{s+1} + \frac{K_2}{s+2} \rightarrow \begin{array}{l} 2 = K_1(s+2) + K_2(s+1) \\ 2 = (K_1+K_2)s + (2K_1+K_2) \end{array}$$

$$\begin{array}{l} K_1 + K_2 = 0 \\ 2K_1 + K_2 = 2 \end{array} \rightarrow \begin{array}{l} K_1 = 2 \\ K_2 = -2 \end{array}$$

W/ ROOTS REAL & REPEATED:

$$F(s) = \frac{2}{(s+1)(s+2)^2} = \frac{K_1}{s+1} + \frac{K_2}{(s+2)^2} + \frac{K_3}{(s+2)} \rightarrow \text{RESIDUALS:}$$

MULT. BY $s+1$

$$\frac{2}{(s+2)^2} = K_1 + \frac{(s+1)K_2}{(s+2)^2} + \frac{(s+1)K_3}{s+2} \rightarrow s=-1 \rightarrow K_1 = 2$$

K_2: MULT. BY $(s+2)^2$

$$\frac{2}{(s+1)} = \frac{K_1(s+2)^2}{s+1} + K_2 + (s+2)K_3 \rightarrow K_2 = -2$$

K_3: DIFFERENTIATE

$$\frac{-2}{(s+1)^2} = K_3 + \frac{2(s+2)(s+1) - (s+2)^2}{(s+1)^2} \rightarrow s=-2 \;,\; K_3 = -2$$

$$f(t) = \left[2e^{-t} - 2te^{-2t} - 2e^{-2t}\right]u(t)$$

W/ COMPLEX ROOTS:

$$F(s) = \frac{s}{(s-1)(s^2+1)} = \frac{K_1}{s-1} + \frac{K_2 s + K_3}{s^2+1} \rightarrow \text{MULT. BY } s-1:$$

$$\frac{s}{s^2+1} = K_1 + (s-1)\left(\frac{K_2 s + K_3}{s^2+1}\right) \rightarrow s=1 \rightarrow \frac{1}{1+1} = K_1 = 1/2$$

$$K_2, K_3 \rightarrow s = K_1(s^2+1) + (s-1)(K_2 s + K_3) \Rightarrow s = (K_1+K_2)s^2 + (K_3-K_2)s + K_1 - K_3$$

$$K_1 + K_2 = 0 \;,\; K_3 - K_2 = 1 \;,\; K_1 - K_3 = 0 \rightarrow K_2 = 1/2 \;,\; K_3 = 1/2$$

$$F(s) = \frac{1/2}{s-1} + \frac{1/2 - 1/2 s}{s^2+1} \rightarrow \boxed{\left[\tfrac{1}{2}e^t + \tfrac{1}{2}\sin t + \tfrac{1}{2}\cos t\right]u(t)}$$

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END TO END EXAMPLE:

$$\left.\begin{aligned} \dot{x} - y &= e^t \\ \dot{y} + x &= \sin t \end{aligned}\right\} \text{ w/ } x(0)=1,\ y(0)=0$$

LT:

1: $sX(s) - x(0) - Y(s) = \frac{1}{s-1} \rightarrow sX(s) - Y(s) = \frac{1}{s-1} + 1$

2: $sY(s) - y(0) + X(s) = \frac{1}{s^2+1} \rightarrow X(s) + sY(s) = \frac{1}{s^2+1}$

→ MULT. EQ. 1 BY s ←

$$s^2X(s) + X(s) = \frac{s}{s-1} + s + \frac{1}{s^2+1} \rightarrow X(s) = \frac{s}{(s-1)(s^2+1)} + \frac{s}{(s^2+1)} + \frac{1}{(s^2+1)^2}$$

→ MULT. EQ. 2 BY −s ←

$$-sX(s) - s^2Y(s) = \frac{-s}{s^2+1} \quad \ldots \rightarrow -s^2Y(s) - Y(s) = \frac{1}{s-1} + 1 - \frac{s}{s^2+1}$$

$$Y(s) = \frac{-1}{(s^2+4)(s-1)} - \frac{1}{s^2+1} + \frac{-s}{(s^2+1)^2}$$

???

10-10-2013

BEABOX.ORG → DANCING ROBOTS

GOALS: TRANSFER FUNCTIONS
BLOCK DIAGRAMS BASICS

LOOKING AT TIME-INVARIENT SYSTEMS

→ CAN'T ANALYZE ROCKET W/ FUEL

TRANSFER FUNCTION:

SYS. W/ INPUT $x(t)$, OUTPUT $y(t)$

$\ddot{y} + 2\dot{y} + y = x(t)$ → TIME DOMAIN

$$s^2Y(s) + 2sY(s) + Y(s) = X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2+2s+1} = G(s)$$

10-10-2013

CAREFUL:

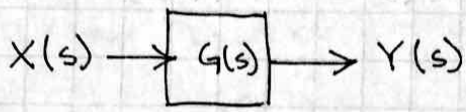
$$Y(s) = G(s)X(s)$$

$Y(s)$ ← OUTPUT, $G(s)$ ← PLANT, $X(s)$ ← INPUT

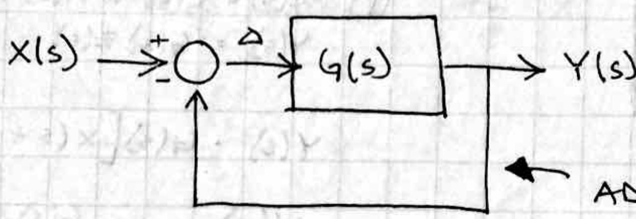
GOOGLE "VISUAL AID FOR LAPLACE TRANSFORMS"

* BLOCK DIAGRAMS:

→ GRAPHICAL REP. OF TRANSFER FUNCTION:



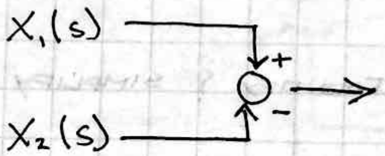
WANT TO REPRESENT CHANGES IN SYSTEM



ADDITION OF FEEDBACK LOOP MAKES AN OPEN SYSTEM A CLOSED SYSTEM

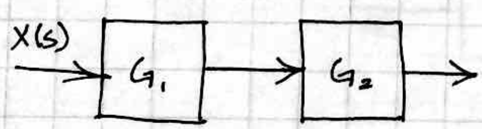
* BASIC BLOCK DIAGRAM ALGEBRA

(1.)



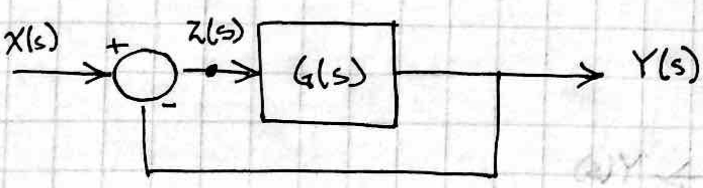
$\rightarrow Y(s) = X_1(s) - X_2(s)$

(2.)



$\rightarrow Y(s) = G_1(s)G_2(s)X(s)$

(3.)



$$Y(s) = G(s)\,Z(s)$$
$$Z(s) = X(s) - Y(s)$$

$$\rightarrow Y(s) = G(s)\left[X(s) - Y(s)\right] \rightarrow Y(s) + G(s)Y(s) = G(s)X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)}$$

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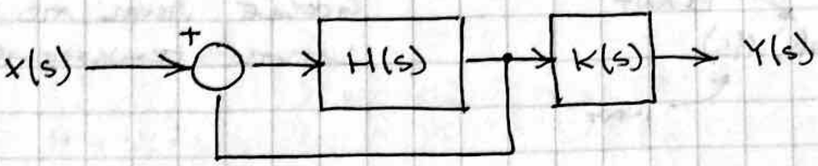
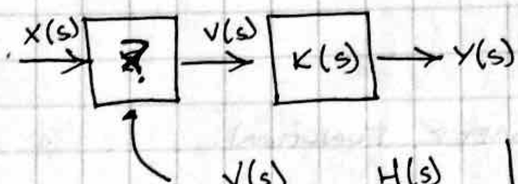
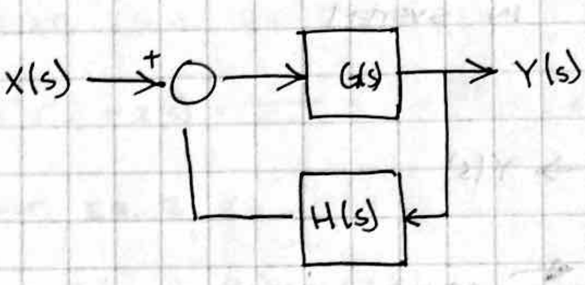


WHAT IS THE TRANSFER FUNCTION?

$$\frac{Y(s)}{X(s)} \quad \begin{matrix}\text{OUTPUT}\\ \text{INPUT}\end{matrix}$$



$$\frac{V(s)}{X(s)} = \frac{H(s)}{1+H(s)} \quad \Bigg| \quad \frac{Y(s)}{X(s)} = \frac{K(s)H(s)}{1+H(s)}$$



(1) $Z(s) = X(s) + H(s)Y(s)$

$Y(s) = G(s)Z(s)$

$$Y(s) = G(s)\left[X(s + H(s)Y(s)\right]$$

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 - G(s)H(s)}$$

* BLOCK DIAGRAM REDUCTION:

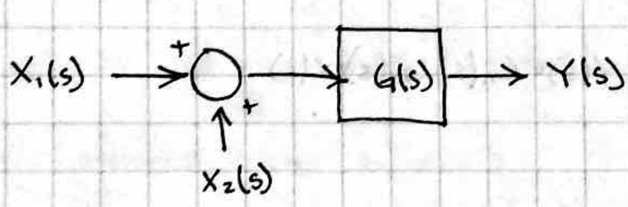
→ WANT TO SIMPLIFY

→ TOOLS → SERIES

↳ FEEDBACK

IDEA → IDENTIFY & SIMPLIFY

TRICKS:



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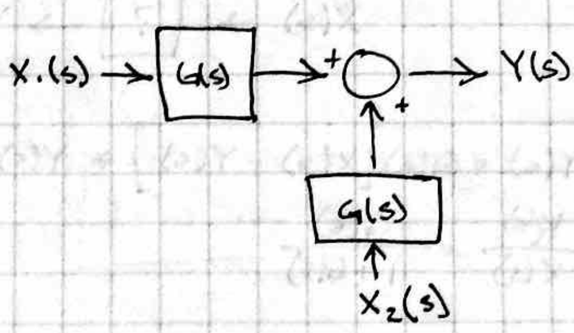

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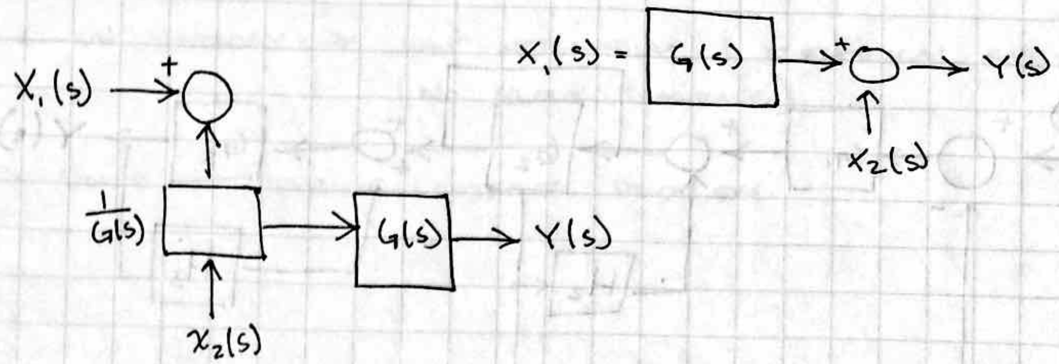
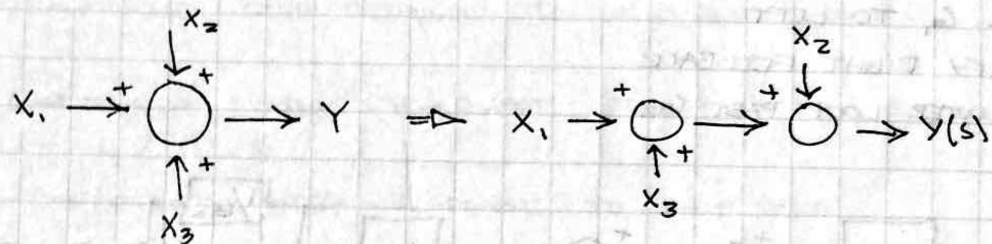
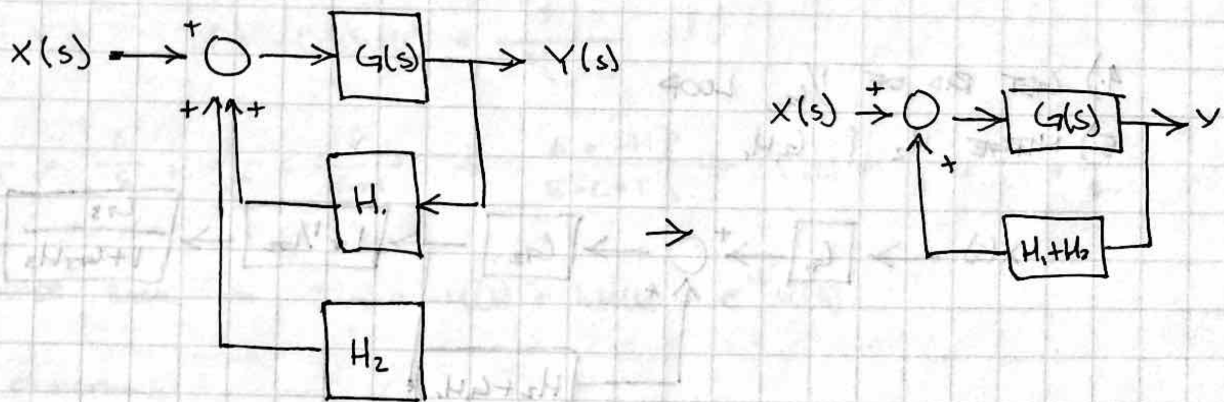


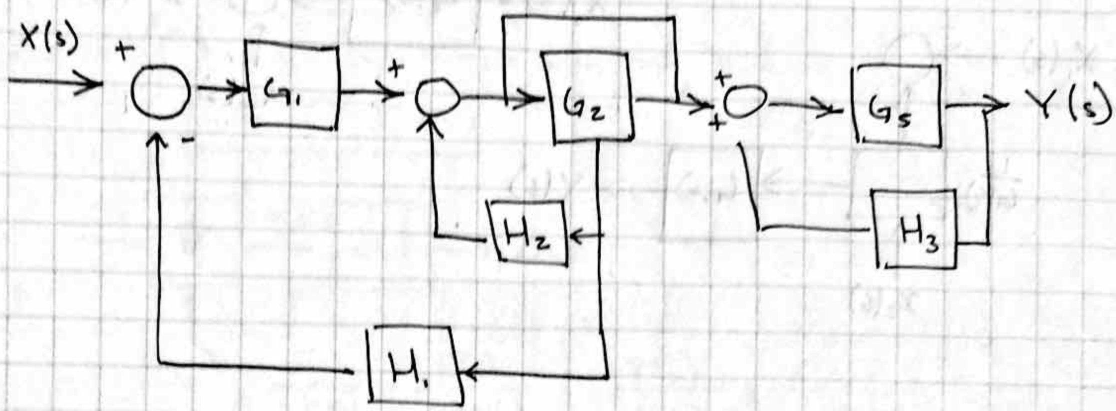
(2.) SUMMING JUNCTIONS:



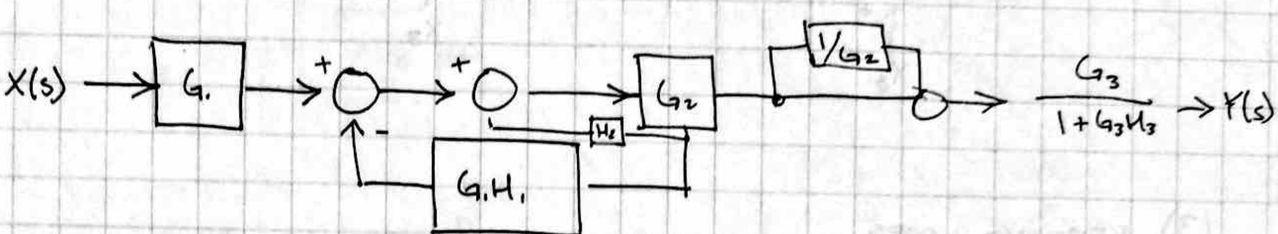
(3.) REMOVING LOOPS



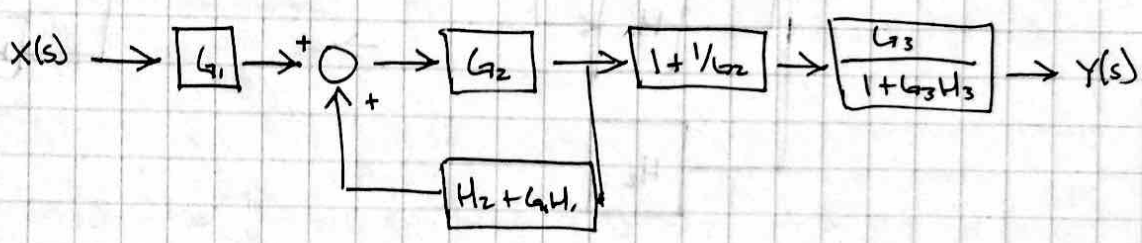
10/10/2013 EXAMPLE 5.2 IN BOOK



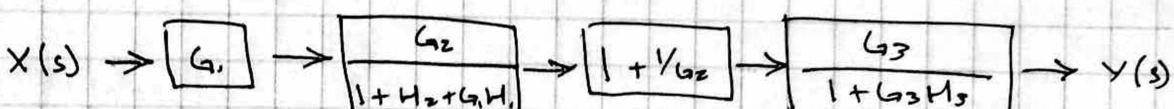
1.) PULL G_1 TO LEFT
2.) SIMPLIFY RIGHT FEEDBACK
3.) MOVE OUTER LOOP PAST G_2



4.) GET RID OF $1/G_2$ LOOP
5.) MERGE H_2 & G_1H_1



6.) TAKE OUT LAST FEED BACK LOOP

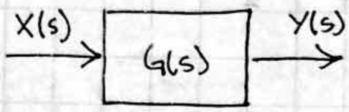


$$\rightarrow G_1 \left[\frac{G_2+1}{G_2}\right]\left[\frac{G_2+1}{G_2}\right]\left[\frac{G_3}{1+G_3H_3}\right]\left[\frac{G_2}{1+[H_2+H_1G_1]G_2}\right] \rightarrow Y(s)$$

10/15/2013:

QUIZ 2 ON THURSDAY → JUST MECHANICAL & ELECTRICAL SYSTEMS (NO BLOCK DIAGRAMS)

NOTES: SOLVE A FEEDBACK CONTROLER PROBLEM



$$G(s) = \frac{1}{s(s+1)}$$

1.) FIND UNCONTROLLED STEP RESPONSE TO UNIT STEP INPUT

2.) FIND CONTROLLED SYSTEM TRANSFER FUNCTION FOR FEEDBACK

3.) FIND CONTROLLED STEP RESPONSE TO UNIT STEP

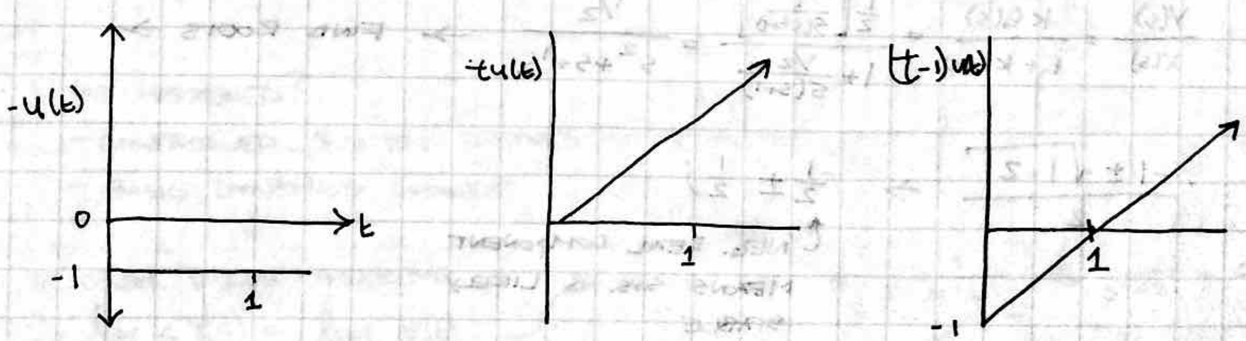
(1.) UNIT STEP INPUT $u(t)$

$$u(s) = \frac{1}{s}, \quad Y(s) = G(s)u(s) = \frac{1}{s^2(s+1)}$$

$$\frac{1}{s^2(s+1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} \rightarrow \left.\begin{matrix} A = -1 \\ B = C = 1 \end{matrix}\right\} \rightarrow Y(s) = \frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}$$

→ TRANSFER BACK → $y(t) = -u(t) + tu(t) + e^{-t}u(t)$

SKETCH FUNCTIONS:



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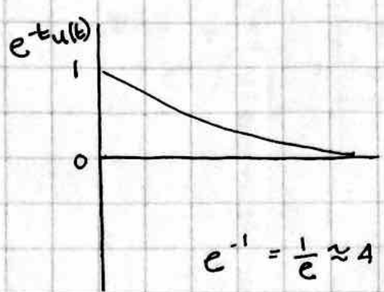
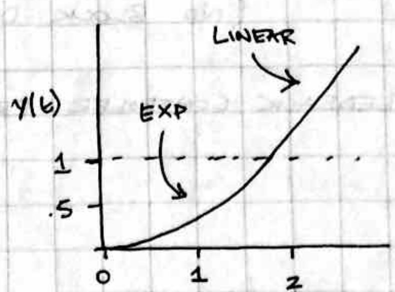


$e^{-1} = \frac{1}{e} \approx .4$

$e^{-2} = \frac{1}{e^2} \approx .15$

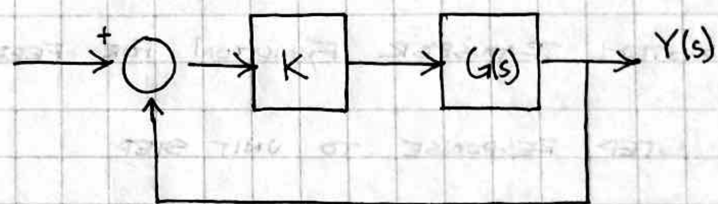


$t = 1, \quad y(t) \approx .4$

$t = 2, \quad y(t) \approx 1.15$

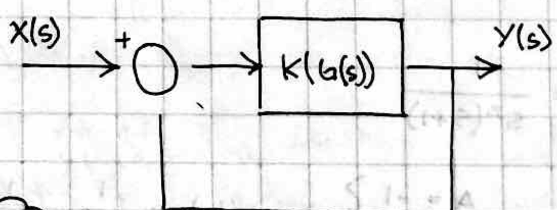
$t = 3, \quad y(t) \approx 2$

(2.)



– K IS CONSTANT (CALLED A PROPORTIONAL CONTROLLER)

– FEEDBACK CONTROL



$K = \frac{1}{2}$

$Y(s) = Z(s)KG(s)$

$Z(s) = X(s) - Y(s)$

$Y(s) = [X(s) - Y(s)]KG(s)$

$Y(s)[1 + KG(s)] = X(s)KG(s) \rightarrow \frac{Y(s)}{X(s)} = \frac{KG(s)}{1 + KG(s)}$

$$\frac{Y(s)}{X(s)} = \frac{KG(s)}{1 + KG(s)} = \frac{\frac{1}{2}\left[\frac{1}{s(s+1)}\right]}{1 + \frac{1/2}{s(s+1)}} = \frac{1/2}{s^2 + s + 1/2}$$ → FIND ROOTS →

$$\frac{-1 \pm \sqrt{1-2}}{2} \rightarrow \frac{-1}{2} \pm \frac{1}{2}j$$

NEG. REAL COMPONENT MEANS SYS. IS LIKELY STABLE

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$$\frac{1/2}{s^2+s+1/2}\,u(s) = \frac{1/2}{s^2+s+1/2}\cdot\frac{1}{s} \rightarrow \frac{A}{s} + \frac{Bs+C}{s^2+s+1/2} \rightarrow \begin{cases} A = 1 \\ B = C = -1 \end{cases}$$

$$Y(s) = \frac{1}{s} + \frac{(-s-1)}{s^2+s+1/2} = \underbrace{\frac{1}{s} - \frac{s+1}{s^2+s+1/2}}$$

$$\frac{s+a}{(s+a)^2+\omega^2} = e^{-at}\cos\omega t$$

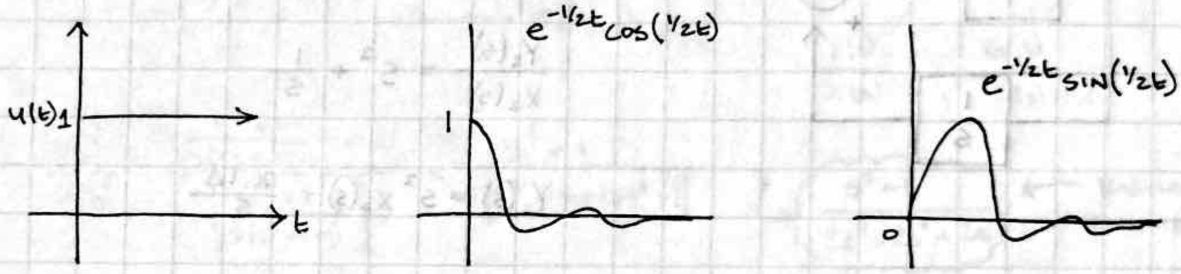
$$\frac{\omega}{(s+a)^2+\omega^2} = e^{-at}\sin\omega t$$

$$\downarrow$$

$$\frac{1}{s} - \frac{(s+1/2)+1/2}{(s+1/2)^2+1/4} = \frac{1}{s} - \frac{(s+1/2)}{(s+1/2)^2+1/4} + \frac{1/2}{(s+1/2)^2+1/4}$$

$$\mathcal{L}^{-1}\{:\} \Rightarrow y(t) = \left[1 - e^{-1/2t}\cos(1/2t) - e^{-1/2t}\sin(1/2t)\right]u(t)$$

→ SKETCH ←



COMBINED SYSTEM RESPONSE:

STABLE (CONVERGES TO VALUE OF UNIT STEP)

WHAT HAPPENED?
- CONTROLLED SYSTEM BEHAVES DIFFERENTLY
- BASIC DYNAMICS CHANGE

* FINAL VALUE THEORUM:

$$\lim_{s\to 0} sY(s) = \lim_{t\to\infty} y(t)$$

UNCONTROLED: $\cancel{s}\left[\frac{1}{\cancel{s}}\frac{1}{s(s+1)}\right] \rightarrow \frac{1}{s^2+s} = \infty$

CONTROLLED: $s\left[\frac{1}{s}\frac{1/2}{s^2+s+1/2}\right] = \frac{1/2}{1/2} = 1$ ← CONTROLLED!

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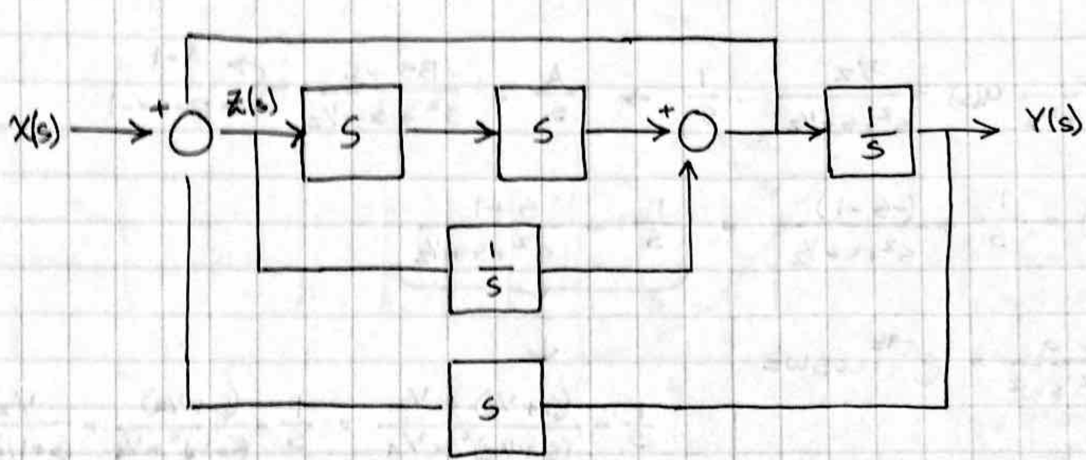
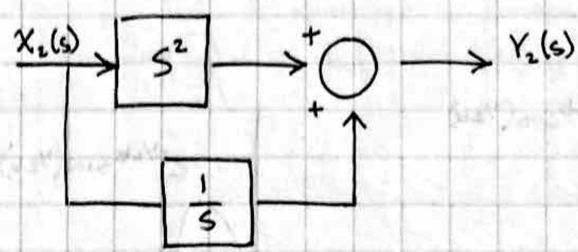
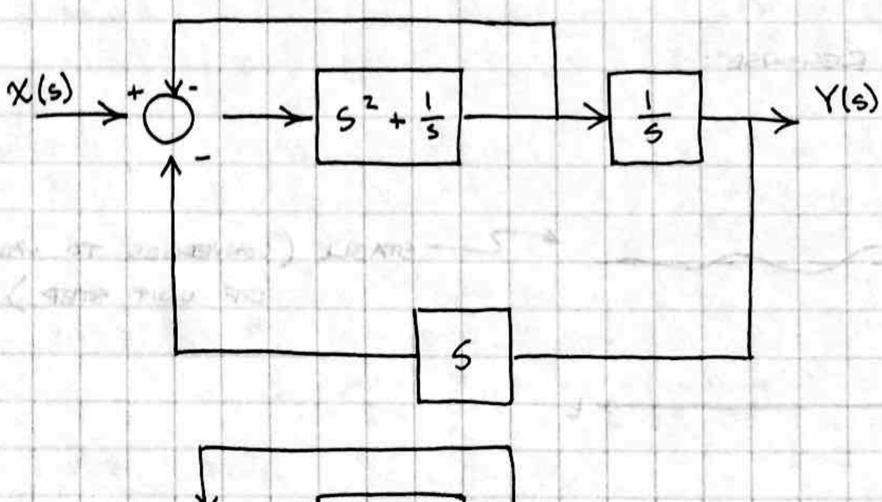
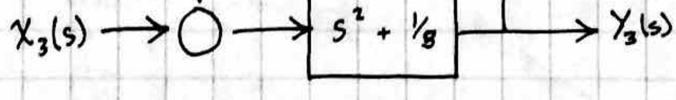




$$\frac{Y_2(s)}{X_2(s)} = S^2 + \frac{1}{S}$$

$$Y_2(s) = s^2 x_2(s) + \frac{x_2(s)}{s}$$





$$Y_3(s) = Z(s)G(s)$$

$$Z(s) = [X_3(s) - Y_3(s)]$$

$$Y_3(s) = G(s)[X_3(s) - Y_3(s)]$$

$$\frac{Y_2(s)}{Y_3(s)} = \frac{G(s)}{1 + G(s)} \rightarrow \frac{s^2 + 1/s}{1 + s^2 + 1/s} = \frac{s^3 + 1}{s^3 + s + 1}$$

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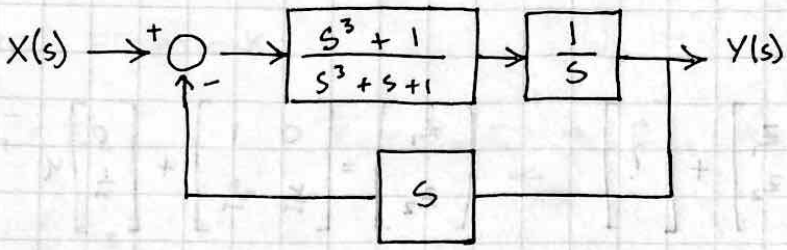

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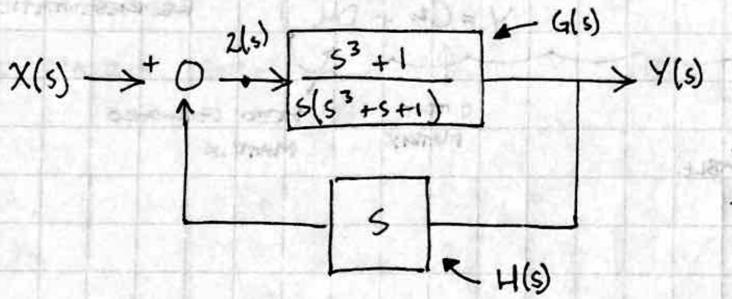

$Y(s) = Z(s)G(s)$

$Z(s) = X(s) - H(s)Y(s)$

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{Y(s)}{X(s)} = \frac{\frac{s^3+1}{s(s^3+s+1)}}{1+\left[\frac{s^3+1}{s(s^3+s+1)}\right]s} = \frac{s^3+1}{s\left[s^3+s+1+s^3+1\right]}$$

$$\frac{s^3+1}{2s^4+2s^3+2s}$$ ← BASICALLY THE END OF THE DAY

* STATE SPACES:

GOALS: REPRESENT SYSTEM IN STATE SPACE REPRESENTATION
- CAST n^{TH} ORDER D.E TO FIRST ORDER FORM

REASON: MOST NUMERICAL METHODS ARE FOR 1^{ST} ORDER FORM ODE'S
- VECTOR / MATRIX FORM MAKES IT EASIER TO MANIPULATE EQUATIONS
  - WORKS FOR NON-LINEAR

$m\ddot{y} + D\dot{y} + Ky = u$ → 2^{ND} ORDER → 2 1^{ST} ORDER ODE'S

$$\left.\begin{array}{ll} z_1 = y, & z_1' = \dot{y} = \dot{z}_2 \\ z_2 = \dot{y} & \dot{z}_2 = \ddot{y} \end{array}\right\} m\dot{z}_2 + Dz_2 + Kz_1 = u$$

$$\dot{z}_2 + \frac{D}{m}z_2 + \frac{K}{m}z_1 = \frac{u}{m}$$

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→ VECTOR / MATRIX FORM

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} ? \\ ? \end{bmatrix}}_{\text{SYSTEM MATRIX}} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \underbrace{\begin{bmatrix} ? \end{bmatrix}}_{\text{INPUT MATRIX}} \rightarrow \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{D}{m} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

$\dot{z} = Az + Bu$ $\qquad y = Cz + Du$ ← STATE SPACE REPRESENTATION

$\dot{z}$: $\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix}$ → DER. OF STATE VECTOR

z: $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ → STATE VECTOR

u: INPUT VARIABLE

C: OUTPUT MATRIX

D: FEED FORWARD MATRIX

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$$\boxed{\begin{aligned} \dot{z} &= Az + Bu \\ y &= Cz + Du \end{aligned}}$$

$m\ddot{y} + D\dot{y} + ky = u$ → n^{TH} ORDER D.E → n 1^{ST} ORDER EQUATIONS

IN GENERAL:

$$\frac{d^n y(t)}{dt^n} + a_n \frac{d^{n-1} y(t)}{dt^{n-1}} + a_{n-1} + \ldots + a_2 \frac{dy}{dt} + a_1 y(t) = u(t)$$

① STATE VARIABLES

$z_1 = y(t)$

$z_2 = y'(t) = z_1'$

$z_n = \ldots \dfrac{d^{n-1} y(t)}{dt^{n-1}}$

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STATE EQUATIONS:

$$\dot{z}_1 = \dot{z}_2 \qquad \dot{z}_n = -a_n z_n - a_{n-1} z_{n-1} - a_2 z_2 - a_1 z_1 + u(t)$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_{n-1} = z_n$$

VECTOR MATRIX FORM:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \vdots \\ \dot{z}_{n-1} \\ z_n \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & 1 & \cdots & & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_2 & -a_3 & & -a_1 \end{bmatrix}} \begin{Bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{Bmatrix} + \overset{B}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}} u$$

→ CONVERT TRANSFER FUNCTION TO S.S. ←

$$G(s) = \frac{10}{s^3 + 9s^2 + 6s + 4}, \quad Y(s) = G(s)U(s)$$

$$\text{I.L.T} \rightarrow (s^3 + 9s^2 + 6s + 4)Y(s) = 10u(s) \rightarrow \dddot{y} + 9\ddot{y} + 6\dot{y} + 4y = 10u$$

$$\begin{aligned} z_1 &= y \\ z_2 &= \dot{y} \\ z_3 &= \ddot{y} \end{aligned} \rightarrow \begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \end{aligned} \qquad z_3 = -4z_1 - 6z_2 - 9z_3 + 10u$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -6 & -9 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u$$

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$$\left.\begin{array}{l}\ddot{x} + 8\dot{x} + y = 9u \\ \ddot{y} + 3\dot{y} + 12y = 0\end{array}\right\}$$ 2 2^{ND} ORDER DE'S → 4 1^{ST} ORDER DE'S → 4 STATE VARIABLES

$z_1 = x$ $\quad$ $z_3 = y$

$$\boxed{z_2 = \dot{x} = \dot{z}_1 \qquad z_4 = \dot{y} = \dot{z}_3}$$

→ PLUG INTO INITIAL EQN'S

$$\dot{z}_2 + 8z_2 + z_3 = 9u$$
$$\dot{z}_4 + 3z_4 + 12z_3 = 0$$

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\\ \dot{z}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -8 & -1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & -12 & -3\end{bmatrix}\begin{Bmatrix}z_1\\ z_2\\ z_3\\ z_4\end{Bmatrix} + \begin{bmatrix}0\\ 9\\ 0\\ 0\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0 & 5 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\\ z_4\end{bmatrix} + \begin{bmatrix}0\end{bmatrix}u$$

NOW, $\dot{z} = \underset{\uparrow \text{ LINEAR}}{A}z + Bu$, WHAT IF $\dot{z} = f(z,u)$?

$$\left.\begin{array}{l}\ddot{x} + 8\dot{x} + \cos y = 9u \\ \ddot{y} + 3\dot{x}\dot{y} + 12y = 0\end{array}\right\} \rightarrow \begin{array}{ll} z_1 = x & z_3 = y \\ z_2 = \dot{x} = \dot{z}_1 & z_4 = \dot{y} = \dot{z}_3\end{array}$$

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\\ \dot{z}_4\end{bmatrix} = \begin{bmatrix}z_2\\ -8z_2 - \cos z_3 + 9u\\ z_4\\ -3z_2z_4 - 12z_3\end{bmatrix}$$

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CONVERT STATE SPACE TO TRANSFER FUNCTION

$$\left.\begin{aligned} \dot{z} &= Az + Bu \\ y &= Cz + Du \end{aligned}\right\} \quad \begin{aligned} y &= \text{OUTPUT} \\ z &= \text{STATE} \\ u &= \text{INPUT} \end{aligned}$$

$$sZ(s) = A\,z(s) + Bu(s)$$

$$Y(s) = C\,z(s) + Du(s)$$

→ SOLVE FOR $Z(s)$ ←

$$Z(s) = (sI - A)^{-1} Bu(s)$$

$$Y(s) = \big[\,C\,(sI - A)^{-1} B + D\,\big]\,u(s)$$

↳ $T(s)$ TRANSFER FUNCTION AS A MATRIX

EXAMPLE:

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} z + \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix} u \quad \rightarrow \quad Y = [1\,0\,0]\,x$$

$$T(s) = \big[\,(sI - A)^{-1} B + D\,\big]$$

* SIMULINK DEMO

→ MATLAB CW → "SIMULINK" → NEW MODEL

[STEP] → [TRANSFER FUNCTION] → [SCOPE] ← DOUBLE-CLICK FOR SYS. RESPONSE

10/22/2013

QUIZ 3 ON THURSDAY

COMPUTER ASSIGNMENT IS POSTED, DUE NOV. 5TH

* NOTES:

1^{st}, 2^{nd} ORDER SYSTEMS

POLES & ZERO'S → <u>POLES OF TRANSFER FUNC.</u>

VALUES OF S THAT MAKE $G(s) \to \infty$, ROOTS OF DENOM.

<u>ZEROS OF TRANSFER FUNC.</u>

VALUES OF S THAT MAKE $G(s) \to 0$

EXAMPLE:

$$G(s) = \frac{s+3}{(s+2)(s+4)(s+5)} \quad , \quad \text{POLES; } -2, -4, -5 \quad \text{ZERO'S: } -3$$

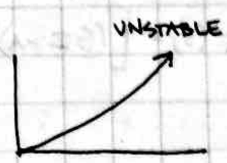
$$G(s) = \frac{K_1}{s+2} + \frac{K_2}{s+4} + \frac{K_3}{s+5} \to \text{NEGATIVE POLES}$$

$$\mathcal{L}^{-1}\{\cdot\} \to K_1 e^{-2t} + K_2 e^{-4t} + K_3 e^{-5t} \to$$

<u>STABLE</u> W/ NEG. POLES

EXAMPLE:

$$G(s) = \frac{1}{s-2} \quad , \quad \text{POLE} \Rightarrow s-2 \to \mathcal{L}^{-1}\{\cdot\} = e^{2t} \to$$



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* FIRST ORDER SYSTEMS *

CONSIDER 1^{st} ORDER SYSTEM W/O ZEROS

→ GENERAL FORM OF TRANSFER FUNCTION $T(s) = \frac{a}{s+a}$

$$\frac{2}{5s+12} = \frac{2/5}{s+12/5}\left(\frac{-5}{12}\right)\left(\frac{2}{5}\right) \to \frac{1}{6}\left[\frac{12/5}{s+12/5}\right]$$

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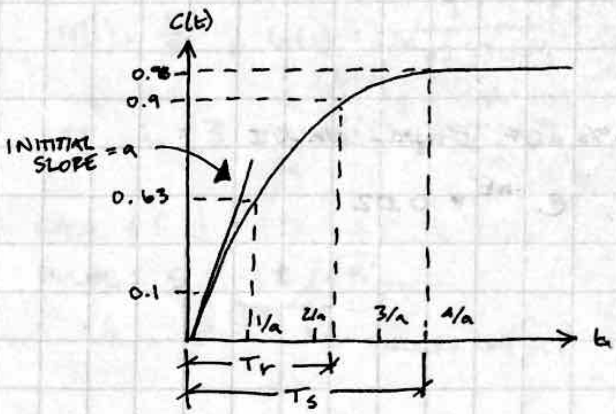
$$Y(s) = T(s)u(s) \rightarrow \frac{a}{s(s+a)} = \frac{K_1}{s} + \frac{K_2}{s+a}$$

$$\mathcal{L}^{-1}\{\cdot\} \rightarrow y(t) = \underbrace{y_f(t)}_{\text{FORCED RESPONSE}} + \underbrace{y_n(t)}_{\text{NATURAL RESPONSE}} = K_1 + K_2 e^{-at} \rightarrow \text{SOLVE FOR } K_1, K_2$$

FORCED → NATURAL ↓

$$= \underbrace{1 - e^{-at}}$$

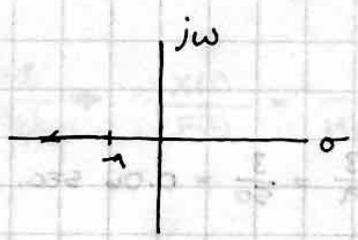
GENERAL SOLUTION FOR 1ST ORDER EQ.



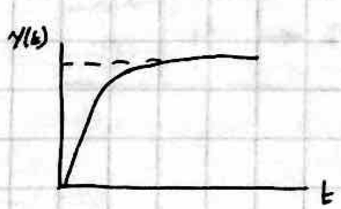
TIME CONSTANTS:

$1/a$ IS TIME CONSTANT OF RESPONSE TIME TO REACH 63% OF FINAL VALUE

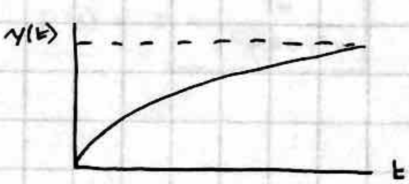
$$e^{-at}\Big|_{t=1/a} = e^{-1} \approx .37, \quad y(1/a) = 1 - .37 = 0.63$$



FARTHER a IS FROM IMAGINARY AXIS, FASTER TRANSIENT RESPONSE



a ↑ TIME CONST. ↓



a ↓ TIME CONST. ↑

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RISE TIME T_r

RISE TIME = TIME FROM .1 TO .9 OF FINAL VALUE

$$y(t) = 1 - e^{-at}, \quad y(t) = 0.1 \rightarrow 1 - e^{-at} = 0.1$$

$$e^{-at} = 0.9, \quad -at = \ln(0.9), \quad t = \frac{0.11}{a}$$

$$\frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a}$$

SETTLING TIME T_s

TIME TO REACH AND STAY WITHIN 2% OF FINAL VALUE

$$y(t) = .98, \quad 1 - e^{-at} = 0.98, \quad e^{-at} = 0.02$$

$$t = \frac{-\ln(0.02)}{a} = \frac{4}{a}$$

EXAMPLE:

$$G(s) = \frac{50}{s+50}, \quad Y(s) = \frac{1}{s} G(s) = \frac{50}{s(s+50)} \rightarrow \mathcal{L}^{-1}\{\cdot\} = 1 - e^{-50t}$$

a.) $\tau = 1/a = 1/50 = 0.02$ SEC.
$T_s = 4/a = 4/50 = 0.08$ SEC.
$T_r = 2.2/a = 2.2/50 = 0.044$ SEC

ALWAYS!

$\tau < T_r < T_s$

→ 5% SETTLING TIME ←

$$0.95 = 1 - e^{-at} \rightarrow t = \frac{-\ln(0.05)}{a} = \frac{3}{a} = \frac{3}{50} = 0.06 \text{ SEC.}$$

10/22/2013 → 2^{ND} ORDER SYSTEMS ←

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

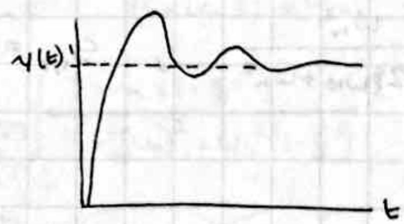
ξ = DAMPING RATIO

ω_n^2 = NATURAL FREQUENCY (W/O DAMPING)

POLES → QUADRADIC FORMULA:

$$s = \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

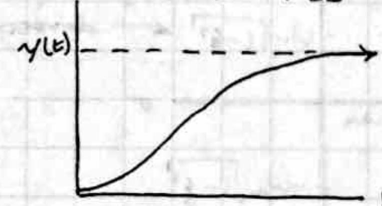
UNDERDAMPED



EXAMPLE:

$$U(s) = \frac{1}{s} \quad , \quad G(s) = \frac{9}{s^2+2\xi 3s+9}$$

OVERDAMPED

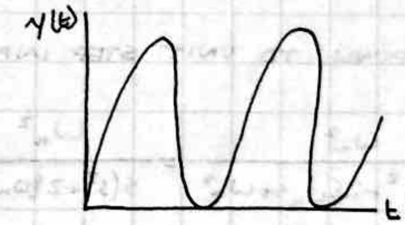


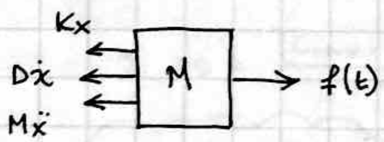
LET $\omega_n = 3 \Rightarrow G(s) = \frac{9}{s^2+6\xi s+9}$

CASE 1:

POLES: $s = \underbrace{-1 \pm j\sqrt{8}}_{}$ ↳ UNDERDAMPED

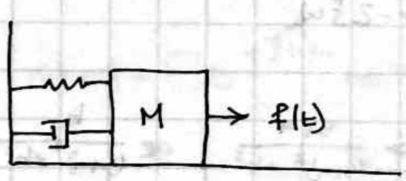
UNDAMPED





→ $(Ms^2 + Ds + k)X(s) = F(s)$

$$\cancel{\frac{F(s)}{X(s)}} \Rightarrow \frac{X(s)}{F(s)} = \frac{1}{Ms^2+Ds+k} = \left[\frac{\frac{1}{M}}{s^2+\frac{D}{M}s+\frac{K}{M}}\right] \cdot \frac{1}{K}$$



$$F(s) \longrightarrow \boxed{\frac{16}{s^2+8s+16}} \longrightarrow X(s)$$

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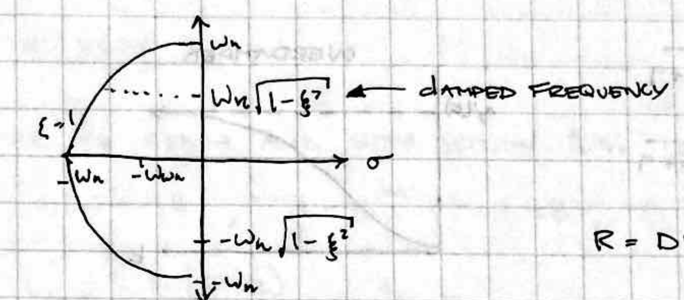
UNDER DAMPED SYSTEMS $\rightarrow$ $\underbrace{0 < \xi < 1}$ $\hookrightarrow$ MEANS COMPLEX ROOTS

$$S = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

$$T(s) = \frac{\omega_n^2}{S^2+2\xi\omega_n s+\omega_n^2}, \quad S_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$



R = DISTANCE FROM POLES TO ORIGIN

$$R = \sqrt{(\xi\omega_n)^2 + \omega_n^2(1-\xi^2)} = \sqrt{\omega_n^2} = \underline{\underline{\omega_n}}$$

SYSTEM RESPONSE TO UNIT STEP INPUT:

$$Y(s) = G(s)u(s)$$

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}$$

$$Y(s) = \frac{K_1}{s} + \frac{K_2 s + K_3}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\omega_n^2 = K_1(s^2+2\xi\omega_n s+\omega_n^2) + K_2 s^2 + K_3 s$$

$$\left.\begin{array}{l} s^2:\ K_1 + K_2 = 0 \\ s^1:\ 2\xi\omega_n K_1 + K_3 = 0 \\ s^0:\ K_1\omega_n^2 = \omega_n^2 = \omega_n^2 \end{array}\right\} \quad \begin{array}{l} K_1 = 1 \\ K_2 = -1 \\ K_3 = -2\xi\omega_n \end{array}$$

So,

$$Y(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2} \rightarrow \text{RECALL} \rightarrow \frac{s+a}{(s+a)^2+b^2}, \text{ OR } \frac{b}{(s+a)^2+b^2}$$

$$Y(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)}$$

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$$Y(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)} - \boxed{}\underbrace{\frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)}}$$

$$\frac{-\xi}{-\sqrt{1-\xi^2}} \quad \frac{\omega_n\sqrt{1-\xi^2}}{(s+\omega_n\xi)^2+\omega_n^2(1-\xi^2)}$$

$$Y(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)} - \left[\frac{\xi}{\sqrt{1-\xi^2}}\right]\frac{\omega_n\sqrt{1-\xi^2}}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)}$$

$$\mathcal{L}^{-1}\{:\} = y(t) = 1 - e^{-\xi\omega_n t}\cos\left(\omega_n\sqrt{1-\xi^2}\,t\right) - \frac{\xi}{\sqrt{1+\xi^2}}e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}\,t\right)$$

→ PERFORMANCE PARAMETERS:

* T_p: DIFFERENTIATE $y(t)$ AND SET TO ZERO

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

* % OVERSHOOT: FIRST FIND C_{MAX}, PLUGE T_P INTO $y(t)$ → $C_{MAX} = 1 + e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$

$$\%\,OS = \frac{C_{MAX} - C_{FINAL}}{C_{FINAL}} \times 100 = \frac{1 + e^{\frac{-\xi\pi}{\{1-\xi^2\}^{1/2}}} - 1}{1} \times 100$$

$$= 100\, e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

* $T_{S_{2\%}}$ AMPLITUDE OF OSCILLATIONS TO REACH 0.02

$$y(t) = 1 - \boxed{1}\, e^{-\xi t\omega_n}\cos(\cdot) - \frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi t\omega_n}\sin(\cdot)$$

$$\text{MAG. OF OSCILLATION} = \left\{\left(e^{-\xi t\omega_n}\right)^2 + \frac{\xi}{\sqrt{1+\xi^2}}\left(e^{-\xi t\omega_n}\right)^2\right\}^{1/2}$$

$$= e^{-\xi t\omega_n}\left\{1 + \frac{\xi}{\sqrt{1-\xi^2}}\right\}^{1/2} \rightarrow \frac{1}{\sqrt{1-\xi^2}}$$

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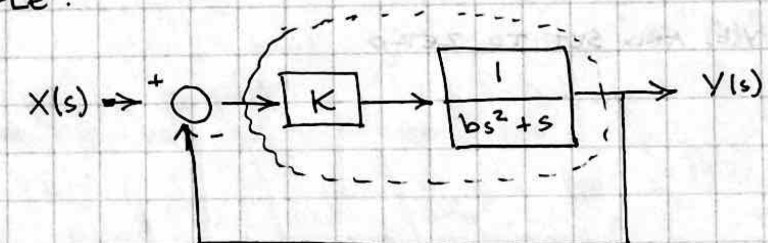
* CONVENTION:

$$T_{s,2\%} = \frac{4}{\xi \omega_n}$$

$$T_{s,1\%} = \frac{4.6}{\xi \omega_n}$$

* RISE TIME:

EXAMPLE:



DESIGN FOR K & b FOR $T_{s,1\%} = 0.25\text{SEC}$

$$\frac{K/bs^2+s}{1 + K/bs^2+s} \rightarrow T(s) = \frac{K}{bs^2+s+K} = \frac{K/b}{s^2 + s/b + K/b}$$

$$\omega_n^2 = \frac{K}{b} \quad (1) \qquad T_s = \frac{4.6}{\xi \omega_n}$$

$$\omega_n = 18.4$$

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$s = \sigma \pm j\omega$

$s = \sigma \pm j\omega \rightarrow e^{-\sigma t} \rightarrow$

- A LTI SYSTEM IS STABLE IF ALL POLES HAVE NEGATIVE & REAL PARTS
  → POLES HAVE ZERO REAL PART, → MARGINALLY STABLE

2 CHECKS:

1.) IF ANY COEFF. IS ZERO, THEN AT LEAST ONE POLE IS <u>NOT ON THE LHP</u>.

2.) IF ANY COEFF. IS NEGATIVE, THEN AT LEAST ONE POLE IS IN RHP → <u>UNSTABLE</u>

$$X(s) \longrightarrow \boxed{\frac{1}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}} \longrightarrow Y(s)$$

ROUTH TABLE

5: a_5 $\quad a_3 \quad a_1$

4: a_4 $\quad a_2 \quad a_0$

3: $b = \dfrac{-\begin{vmatrix} a_5 & a_3 \\ a_4 & a_2 \end{vmatrix}}{a_4}$ $\qquad b_2 = \dfrac{-\begin{vmatrix} a_5 & a_1 \\ a_4 & a_0 \end{vmatrix}}{a_4}$

2: $c_1 = \dfrac{-\begin{vmatrix} a_4 & a_2 \\ b_1 & b_2 \end{vmatrix}}{b_1}$ $\qquad c_2 = \dfrac{-\begin{vmatrix} a_4 & a_0 \\ b_1 & 0 \end{vmatrix}}{b_1}$

1: $d_1 = \dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{c_1}$ $\qquad 0$

0: $\dfrac{-\begin{vmatrix} c_1 & c_2 \\ d_1 & 0 \end{vmatrix}}{d_1}$

SYSTEM IS STABLE IF AND ONLY IF ALL ELEMENTS IN THE FIRST COLUMN ARE POSITIVE

R.T. STEPS:

1.) LABEL ROWS BY POWERS OF S
2.) START W/ COEFF. OF HIGHEST ~~[illegible]~~ ORDER & LIST <u>EVERY OTHER</u> COEFF. IN ROW
3.) NEXT HIGHEST (SKIPPED COEFF.) ROW AND REPEAT IN SECOND ROW
4.) (-1)*(THE DETERMINATE OF "2 ROWS" ABOVE W/ LEFT MOST COLUMN AS FIRST COLUMN, AND COLUMN IMMEDIATELY ~~BELOW~~ TO THE RIGHT AS SECOND COLUMN DIVIDED BY FIRST ENTRY ON ROW ABOVE

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$$\xrightarrow{} \boxed{\dfrac{1}{s^3 + 10s^2 + 31s + 1030}} \rightarrow$$ QUICK CHECKS PASS ✓

s^3 1 31

s^2 $10/10 = 1$ $1030/10 = 103$

s^1: $\dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72$ 0

s^0: $\dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103$

1
1
-72
$+03$

OF SIGN CHANGES = # OF POLES

EXAMPLE:

$$s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$$

s^5: 1 2 11

s^4: 2 4 10

s^3: $c_1 = \dfrac{-\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix}}{2} = \cancel{0} \rightarrow \epsilon$ $c_2 = \dfrac{-\begin{vmatrix} 1 & 11 \\ 2 & 12 \end{vmatrix}}{2} = 6$ 0

s^2: $d_1 = \dfrac{-\begin{vmatrix} 2 & 4 \\ \epsilon & 6 \end{vmatrix}}{\epsilon} = \dfrac{-(12 - 4\epsilon)}{\epsilon} \approx \dfrac{-12}{\epsilon}$

ϵ IS ~~A~~ A "SMALL" NUMBER

0 IN THE FIRST COLUMN LEADS TO UNSTABLE

$$\frac{-\begin{vmatrix} \epsilon & 6 \\ \frac{-12}{\epsilon} & 10 \end{vmatrix}}{\frac{-12}{\epsilon}} = \frac{-10\epsilon - \left(-\frac{12}{\epsilon} \cdot 6\right)}{\frac{-12}{\epsilon}}$$

$$= 6$$

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EXAMPLE: FULL ROW OF ZERO'S

$$s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56$$

s^5: 1 $\quad 6 \quad 8$

s^4: $7/7 = 1$ $\quad 42/7 = 6 \quad 56/8 = 8$

s^4: $0 \quad 0 \quad 0$ ← WHOLE ROW OF ZERO'S

- FORM A POLYNOMIAL W/ ROW ABOVE ROW OF ZERO'S, SKIPPING POWERS. (JUST LIKE WHEN GENERATING RT)
- DIFFERENTIATE THAT ROW
- ENTER THOSE COEFFICIENTS IN ROW OF ZERO'S

AUXILLARY EQUATION

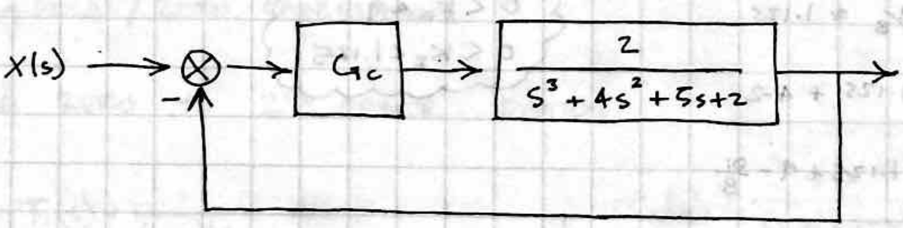
$$s^4 + 6s^2 + 8$$

DIFF: $4s^3 + 12s$

DIVISOR OF ORIGINAL. EQ.

s^3: $4/4 = 1$ $\quad 12/4 = 3 \quad 0$

s^2: $3 \quad 8 \quad 0$



$G_c = K + \frac{K_I}{s}$, $K_P > 0$ $\qquad H(s) = \frac{G_cG_P}{1+G_cG_P} = \frac{(K_P + \frac{K_I}{s})(\quad)}{1+(K_P+\frac{K_I}{s})(\quad)}$

$$= s^4 + 4s^3 + 5s^2 + (2 + 2K_P)s + 2K_I = 0 \text{ , } K_I > 0$$

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4: $1 \quad 5 \quad 2K_I$

3: $4 \quad 2+2K_p \quad 0$

$\cancel{2}: a_1 = \dfrac{-\begin{vmatrix} 1 & 5 \\ 4 & 2+2K_p \end{vmatrix}}{4} = \dfrac{-(2+2K_p-20)}{4} = \dfrac{18-2K_p}{4}$

$a_2 = \dfrac{-\begin{vmatrix} 1 & 2K_I \\ 4 & 0 \end{vmatrix}}{4} = \dfrac{-(8K_I)}{4} = 2K_I$

$\rightarrow$

4: $1 \quad 5 \quad 2K_I$

3: $2 \quad 1+K_p \quad 0$

2: $9-K_p \quad 4K_I \quad 0$

1:

2: $\dfrac{9-K_p}{2} \quad 2K_I$

$1: \dfrac{-\begin{vmatrix} 2 & 1+K_p \\ 9-K_p & 4K_I \end{vmatrix}}{9-K_p} = \dfrac{-(8K_I-(1+K_p)(9-K_p))}{9-K_p}$

$= (1+K_p)(9-K_p) - 8K_I > 0$

$\dfrac{(1+K_p)(9-K_p)}{8} > K_I \rightarrow K_I < \dfrac{9}{8} + \dfrac{8K_p}{8} - \dfrac{K_p^2}{8}$

$K_p = 0; \ K_I < 9/8 \approx 1.125$

$K_p = 4; \ K_I < 1.125 + 4 - 2$

$K_p = 9; \ K_I < 1.125 + 9 - \dfrac{81}{8}$

$0 < K_p < 9$

$0 < K_I < 1.125$

10/30/13 FERIS, ALEX

POLES: ANY VALUE OF 'S' THAT CAUSE THE TERM TO BECOME INFINITE, OR ANY 'S' THAT ~~CAUSES~~ IS SHARED BETWEEN NUM & DENOM.

* OVERDAMPED RESPONSE: $\zeta >$ OVERDAMPED
- TWO REAL ROOTS @ $-\sigma_1, -\sigma_2$ (POLES)

* UNDER DAMPED ($\zeta < 1$)
- TWO COMPLEX @ $-\sigma_d \pm j\omega_d$

$\zeta = 0$ ~~UNDERDAMPED~~

$\zeta < 1$

* UNDAMPED ($\zeta = 0$)
- TWO IMAGINARY @ $\pm j\omega_1$

* CRITICALLY DAMPED ($\zeta = 1$)
- TWO REAL @ $-\sigma_1$

NOTES 10/31

IMPACT OF ADDITIONAL POLES & ZERO'S
- POLE / ZERO CANCELATION

ADD ZERO TO 2ND ORDER SYSTEM

$$T_1(s) = \frac{1}{(s+b)(s+c)} \quad , \quad T_2(s) = \frac{s+a}{(s+b)(s+c)}$$

$\hookrightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$

$$T_1(s) = \frac{1}{(s+b)(s+c)} = \frac{K_1}{s+b} + \frac{K_2}{s+c} \rightarrow 1 = K_1(s+c) + K_2(s+b)$$

s^1 : $K_1 + K_2 = 0$, $K_1 = -K_2$

$s_0 = K_1 c + K_2 b = 1$

$K_1 c - K_1 b = 1$

$K_1(c-b) = 1 \Rightarrow K_1 = \frac{1}{c-b}$

$K_2 = \frac{1}{b-c}$

$$T_2(s) = \frac{s+a}{(s+b)(s+c)} = \frac{K_1}{s+b} + \frac{K_2}{s+c}$$

$s+a = K_1(s+c) + K_2(s+b)$

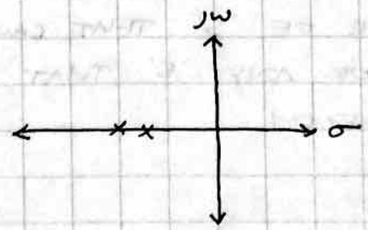
s^1: $K_1 + K_2 = 1$ $\qquad K_2 = 1 - \frac{a-b}{c-b}$

s^0: $K_1 c + K_2 b = a$

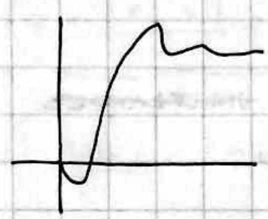
10/31/2013

$$K_2 = 1 - \frac{a-b}{c-b} \rightarrow \frac{c-a}{c-b}$$

$$K_1 = \frac{a-b}{c-b}$$



$$(s+a)\,T_1(s) = sT_1(s) + aT_1(s) = sT_1(s) - aT_1(s)$$



* ADD A POLE :

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}\left[\frac{1}{s+c}\right]$$

$$Y(s) = \frac{A}{s} + \frac{Bs + E}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \frac{D}{s+c} \rightarrow$$

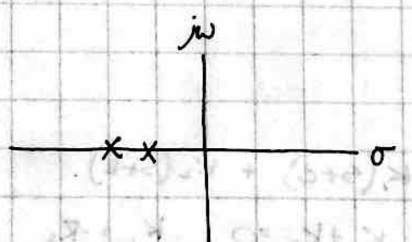
$A = 1$

$$B = \frac{ca - c^2}{c^2 + b - ca} \qquad C = \frac{ca^2 - c^2 a - bc}{c^2 + b - ca}$$

$$d = \frac{-b}{c^2 + b - ca}$$

$$Y(t) = Au(t) + te^{-\zeta\omega_n t}\left[B\cos(\omega_d t) + E\sin(\omega_d t)\right] + De^{-ct}$$

$$s_{1,2} = -\zeta\omega_n \pm j\underbrace{\omega_n\sqrt{1-\zeta^2}}_{\omega_d}$$



AS $c \rightarrow \infty$

$D \rightarrow 0$

$c > 10\times$ OTHER POLES

10/31/2013

LOOK @ SYSTEM: $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

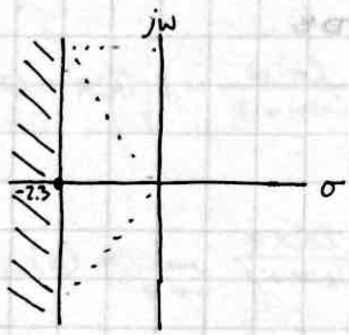
1.) FIND REGION OF ALLOWANCE FOR POLES (S-~~PHASE~~ PLANE), SUCH THAT 1% SETTLING TIME LESS THAN 2 SEC. & OVERSHOOT LESS THAN 10% (STEP INPUT)

2.) SOLVE FOR ξ AND ω_n

$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}\ , \quad T_{s_{1\%}} = \frac{4.6}{\xi\omega_n}$$

$$\frac{4.3}{\xi\omega_n} < 2 \rightarrow \xi\omega_n > 2.3 \rightarrow -\xi\omega_n < -2.3$$

$$\rightarrow \xi\omega_n > 2.3\ , \ \omega_n > 2.3$$



10% OVERSHOOT

$$\%\ OS = 100 e^{-\xi\pi/\sqrt{1-\xi^2}} < 10$$

$$e^{-\xi\pi/\sqrt{1-\xi^2}} < 0.1 \rightarrow \frac{-\xi}{\sqrt{1-\xi^2}} \leq \frac{\ln(0.1)}{\pi}$$

$$\frac{\xi}{\sqrt{1-\xi^2}} > \frac{-\ln(0.1)}{\pi}$$

$$\tan\alpha = \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} \rightarrow \alpha > \tan^{-1}\left[\frac{-\ln(0.1)}{\pi}\right] = 36.24°$$

FIND ξ, ω_n:

WE HAVE $\alpha \rightarrow \tan\alpha > 0.73 \rightarrow \sin\alpha > 0.59$

$$\sin\alpha = \frac{\xi\omega_n}{H} \rightarrow H = \{\xi^2\omega_n^2 + \omega_n^2(1-\xi^2)\}^{1/2}$$

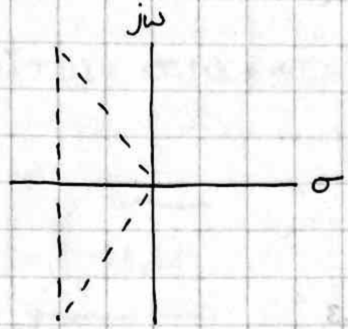
$$H = \sqrt{\omega_n^2} \rightarrow H = \omega_n$$

FROM SETTLING TIME: $\xi\omega_n > 2.3\ , \ \omega_n > \frac{2.3}{\xi}$

10/31/2013

LET $\zeta = 1$, $W_n = 2.3$

$$s = \frac{-4.6}{2} \pm \sqrt{\frac{4.6 - 21.2}{2}} = -2.3 \pm 0$$



* MIDTERM:

- DESIGN & RESPONSE OF 1ST & 2ND ORDER SYSTEMS
- FINAL VALUE THEORUM, PERFERMANCE CHAR.
- STABILITY
- BLOCK DIAGRAM / REDUCTION
- STATE SPACE (CONVERTING TRANS → STATE SPACE)

TOOLS:
- LAPLACE TRANSFORM
- PARTIAL FRAC.
- MECH. & ELECTRICAL SYSTEM

} NO PROBLEM W/ THESE AS SPECIFIC GOALS

NOT ON MIDTERM:
- PID CONTROLERS
- S.S. ERROR

11/3/2013 NOTES:

TIME CONSTANT: $1/a$, OR TIME FOR

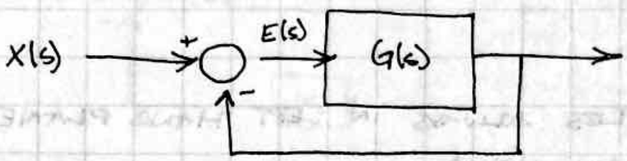
KNOW FINAL VALUE THEORUM FOR MIDTERM

11/5/2013 NOTES (NOT ON MIDTERM)

- S.S ERROR
- P.I.D CONTROL

* STEADY STATE ERROR

- DIFFERENCE BETWEEN INPUT & OUTPUT @ $T \to \infty$



$$E(s) = X(s) - Y(s) \rightarrow Y(s) = \frac{G(s)}{1+G(s)} X(s)$$

$$E(s) = X(s)\left[1 - \frac{G(s)}{1+G(s)}\right] \rightarrow \boxed{E(s) = \frac{X(s)}{1+G(s)}}$$

$$e_{ss}(t) = \lim_{s \to 0} \frac{sX(s)}{1+G(s)}$$

→ IN GENERAL ←

$$X(s) \rightarrow \boxed{T(s)} \rightarrow Y(s) \rightarrow E(s) = X(s) - Y(s)$$

$$Y(s) = T(s)X(s)$$

↳ CLOSED LOOP T.F.

$$E(s) = X(s)\left[1 - T(s)\right] \rightarrow \text{F.V.T } e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} sX(s)\left[1 - T(s)\right]$$

11/5/2013 NOTES:

* EX.

LET C.L T.F:

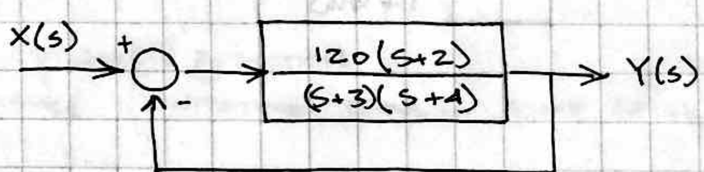
$$T(s) = \frac{5}{s^2+10s+15} \rightarrow X(s) = 1/s$$

$$e_{ss} = \lim_{s\to 0} \frac{s}{s}\left[1 - \frac{5}{s^2+10s+15}\right] \Rightarrow 1 - \frac{5}{15} = \frac{2}{3}$$

→ CHECK F.V.T ←

$$s: \frac{-10 \pm \sqrt{100-60}}{2} \rightarrow$$ POLES ALWAYS IN LEFT HAND PLANE, F.V.T VALID

* EX.



S.S. ERROR FUNC. FOR UNIT STEP

SPECIFIC FOR OPEN LOOP

a.) $x(t) = u(t)$

$X(s) = 1/s$

$$e_{ss} = \lim_{s\to 0} \frac{s\,X(s)}{1+G(s)} = \lim_{s\to 0} \frac{s}{s} \cdot \frac{1}{1 + 240/12} = 1/21$$

→ W/ CLOSED LOOP

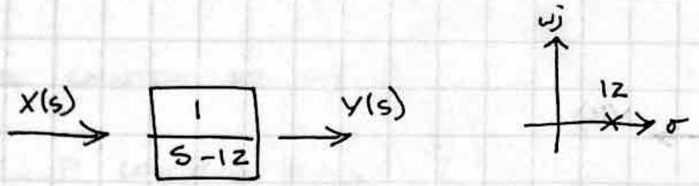
$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{120(s+2)}{(s+3)(s+4)}}{1 + \frac{120(s+2)}{(s+3)(s+4)}} = \frac{120(s+2)}{(s+3)(s+4)+120(s+2)}$$

$$= \frac{120(s+2)}{s^2+127s+252} \rightarrow \lim_{s\to 0} \frac{s}{s}\left[1 - T(s)\right] \rightarrow 1 - \frac{240}{252}$$

$$= \frac{12}{252} = 1/21$$

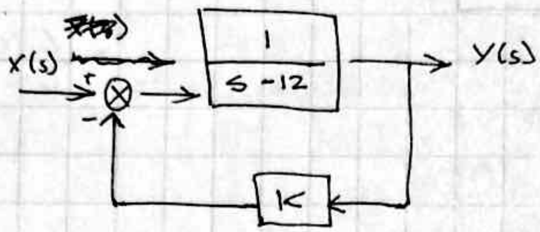
SAME!

11/5/2013



$x(t) = u(t)$ BOUNDED INPUT → BOUNDED OUTPUT? (NO!)

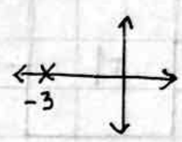
WRAP A CONTROLLER AROUND



CLOSED LOOP T. F:

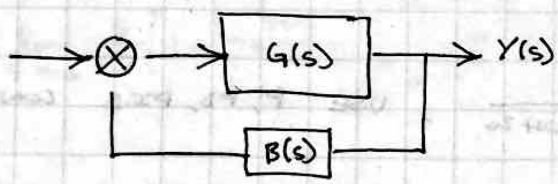
$$\frac{Y(s)}{X(s)} = \frac{1/S-12}{1 + K/S-12} = \frac{1}{S-12+K}$$

→ PICK $\underline{K=15}$ ← $\frac{Y(s)}{X(s)} = \frac{1}{S+3}$



→ BOUNDED OUTPUT:

BASIC CONTROL STRUCTURES



B(s) & F(s) ARE CONTROLLERS

(1) $F(s) = K$ → PROPORTIONAL CONTROLLER

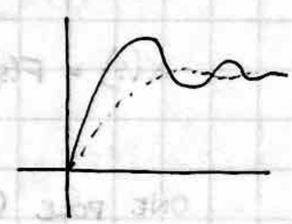
(2) $F(s) = K_D S$ → DERIVATIVE CONTROLLER

(3.) $F(s) = \frac{K_I}{S}$ → INTEGRAL CONTROLLER

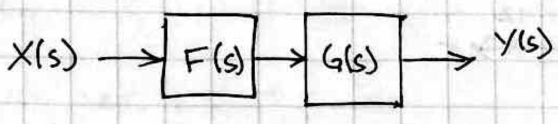
(4.) $K_P + K_D S$ → PD CONTROLLER

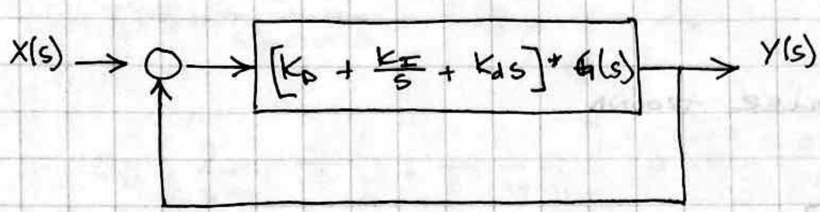
(5.) $K_P + \frac{K_I}{S}$ → PI CONTROLLER

(6.) $K_P + \frac{K_I}{S} + K_D S$ → PID CONTROLLER

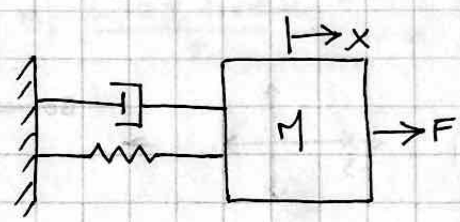


11/5/2013 NOTES:





EXAMPLE:



$$M\ddot{x} + b\dot{x} + Kx = F(t)$$

$$\rightarrow \mathcal{L} \rightarrow (Ms^2 + bs + K)X(s) = F(s) \rightarrow \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Bs + K}$$

KNOWNS:

$M = 1\,kg$

$B = 10\ N{\cdot}s/m$

$K = 20\ N/m$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 20}$$, USE P, PD, PID CONTROL

→ FIRST, OPEN LOOP RESPONSE TO UNIT STEP

$$X(s) = F(s)\left(\frac{1}{s^2+10s+20}\right) \; ; \; F(s) = 1/s = 1/s\left(\frac{1}{s^2+10s+20}\right)$$

ONE POLE @ ORIGIN, POLES @ $10 \pm \sqrt{100-80}$ → IN LHP

$$\omega_n^2 = 20 \rightarrow \omega_n = \sqrt{20}, \quad 2\zeta\omega_n = 10 \rightarrow \zeta = \frac{5}{\sqrt{20}} = \sqrt{\frac{25}{20}} \quad (\zeta > 1)$$

$$x(\infty) = \lim_{s\to 0} sX(s) = \lim_{s \to 0} \frac{1}{s^2+10s+20} = 1/20$$

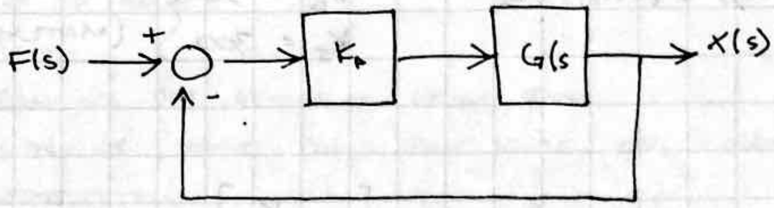
$$e_{ss} = 1 - 1/20 = 19/20$$

* ONLY GETTING US TO 1/20 OF DESIRED OUTCOME *

11/5/2013

→ ADD CONTROLLER ←

* USE P CONTROL *



$$\frac{X(s)}{F(s)} = \frac{K_p}{s^2+10s+(20+K_p)} \rightarrow \text{PICK } K_p = 300, \quad \omega_n^2 = 320$$

$\omega_n = 17.9$

$2\zeta\omega_n = 10$

$\zeta = \frac{10}{2(17.9)} = 0.26 < 1$ → UNDERDAMPED!

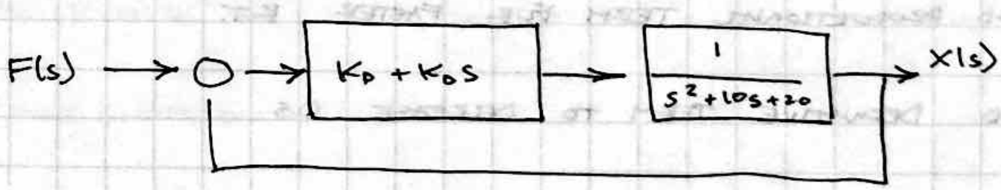
$$T_{s\,2\%} = \frac{4}{\zeta\omega_n} = \frac{4}{5} = 0.8 \text{ SEC}$$

$$T_r = \frac{1.3}{\omega_n} = \frac{1.3}{17.9} \approx 0.07 \text{ SEC}$$ ← REALLY FAST!!

→ BUT, OVERSHOOT: $\approx 40\%$ ← VERY HIGH!

$$e_{ss} = \lim_{s \to 0} \frac{s}{s}\left[1 - \frac{300}{s^2+10s+320}\right] \rightarrow 1 - \frac{300}{320} = \frac{1}{16}$$ ← PRETTY LOW

* USE PD CONTROL *



$$\frac{X(s)}{F(s)} = \frac{K_D s + K_P}{s^2+(10+K_D)s+(K_P+20)}$$ LET $K_P = 300$, $K_D = 10$

$\omega_n^2 = K_P + 20 = 320 \rightarrow \omega_n = 17.9$

$\zeta = 0.6 < 1$ UNDERDAMPED, $\%\,OS = 100\,e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} = 15\%$

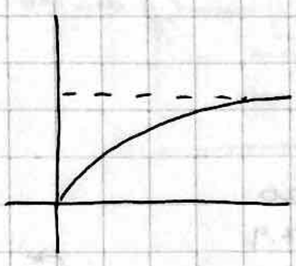
$e_{ss} = 1 - \frac{300}{320} = \frac{1}{16}$ (SAME) SS. ERROR SAME, BUT LESS O.S.

11/5/2013

→ USE PID CONTROL:

$$\frac{X(s)}{F(s)} = \frac{K_D s^2 + K_P s + K_I}{s^3 + (10+K_D)s^2 + (20+K_P)s + K_I}$$

$K_P = 350$
$K_D = 50$
$K_I = 300$
} TRIAL & ERROR (MATLAB)



$$e_{ss} = \lim_{s\to 0} \ldots \left[1 - \frac{K_I}{K_I}\right] = 0$$

NO O.S. NO S.S. ERROR, FAST RISE TIME

C.L	RT	OS	ST	SS. ERROR
K_P	↘	↗	~	↘
K_I	↘	↗	↗	ELIMINATE
K_D	~	↘	↘	~

(GENERAL TRENDS

P.I.D RECIPE:

1.) LOOK @ OPEN LOOP RESPONSE TO SEE WHAT NEEDS IMPROVEMENT

2.) ADD PROPORTIONAL TERM FOR FASTER R.T.

3.) ADD DERIVATIVE TERM TO DECREASE O.S

4.) ADD INTEGRAL TERM TO ELIMINATE S.S. ERROR

5.) KEEP AS SIMPLE AS POSSIBLE

① $R(s) = \frac{A}{s}$

② $R(s) = \frac{A}{s^2}$ $\quad e_{ss} = \lim_{s\to 0} s \frac{A}{1+G(s)}$, $\quad G(s) = \frac{K \prod_i (s+z_i)}{s^N \prod_j (s+p_j)}$

③ $R(s) = \frac{A}{s^3}$

11/9/2013

CONTROLS PROJECT QUESTIONS

HAVE A COUPLE REFERENCES

- SCOPE OF PROJECT
  - # OF PAGES
  - DEPTH
  - CAN WE USE EXAMPLES FROM BOOK
  - CAN WE START FROM TIME SPACE, OR S-SPACE (WHERE TO START) YES!
  - EXPECTATIONS

11/12/2013:

- COMPUTER ASSIGNMENT 2 (DUE TWO WEEKS) → POSTED
- HW 6 OUT, HW 7 TO COME
- NO QUIZ THIS WEEK, LONGER QUIZ THIS WEEK

PROPER PROBLEM FORMULATION ← 1 PAGE → INPUT, OUTPUT, SYSTEM
MATLAB
CONTROLLER DESIGN ≈1 PAGE
~5 PAGES

↳ WHAT IS THE GOAL?
↳ CONTROLLER DESIGN
↳ MATLAB IMPLIMENTATION ≈ 1 PAGE
↳ PLOTS SHOWING IF CONTROLLER IS ACHIEVING GOAL

GOAL: MINIMIZE AMOUT OF SUFFARINGS FOR A GIVEN SPACIAL ACCURACY

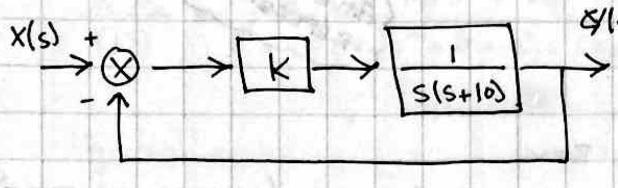
PURPOSE: EXPENSIVE

PROJECT OUTLINE:
1

INPUT → COMP. SIGNAL →
OUTPUT

11/14/2013

* ROOT LOCUS *



$\rightarrow$ C.L $T(s) = \dfrac{K}{s^2 + 10s + K}$

$\rightarrow$ POLES? $\rightarrow$ $s_{1,2} = \dfrac{-10 \pm \sqrt{100-4K}}{2}$

$\rightarrow K = 0 \leftarrow$

$s_1 = 0, \; s_2 = -10$

$\rightarrow K = 9 \leftarrow$

$s_1 = -5 \pm 4 \rightarrow s_1 = -1, s_2 = -9$

$\rightarrow K = 25 \leftarrow$

$s_1 = s_2 = 5$

ALWAYS (AT LEAST MARGINALLY) STABLE,
SETTLING REMAINS CONSTANT FOR $K > 25$
(REAL TERM CONSTANT)

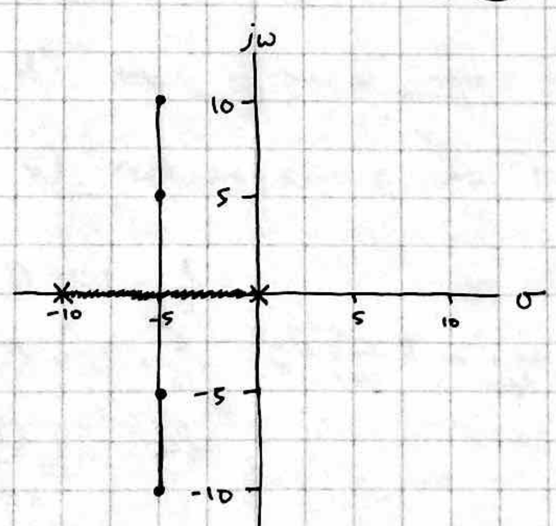
$\rightarrow K = 50 \leftarrow$

$s_{1,2} = -5 + 5j$

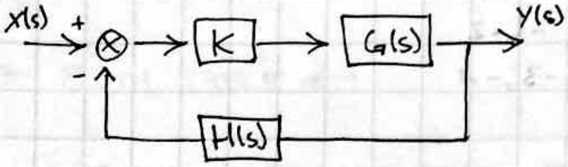
$\rightarrow K = 125 \leftarrow$

$s_{1,2} = -5 \pm 10j$

POLES MOVE AROUND WITH K!



11/14/2013



$\rightarrow$ C.L $\rightarrow Y(s) = \dfrac{K(G(s))}{1 + K\,G(s)H(s)}$

ROOT LOCUS RULES IN BOOK / ONLINE

* INFINITE POLES & ZEROS *

A FUNCTION F HAS A POLE @ ∞ IF: $F \to \infty$ AS $s \to \infty$

A FUNCTION F HAS A ZERO @ ∞ IF: $F \to 0$ AS $s \to \infty$

$F(s) = s$, ZERO: $s = 0$, POLE @ ∞

$F(s) = 1/s$, POLES @ 0, ZERO @ ∞

EVERY FUNCTION HAS AN EQUAL # OF POLES & ZEROS

* ASYMPTOTIC BEHAVIOR

ROOT LOCUS APPROACHES STRAIGHT LINES AS $K \to \infty$

REAL AXIS INTERCEPT & ANGLE OF ASYMPTOTES

$$\sigma_A = \frac{\Sigma \text{ FINITE POLES} - \Sigma \text{ FINITE ZEROS}}{\#\text{ FINITE POLES} - \#\text{ FINITE ZEROS}} \rightarrow \theta_A = \frac{(2n+1)\pi}{\#\text{F.P} - \#\text{F.Z}}$$

EX.

$$T(s) = \frac{K}{s^2 + 10s + K} \rightarrow G(s) = \frac{1}{s(s+10)}, \quad H(s) = 1 \rightarrow G(s)H(s) = \frac{1}{s(s+10)}$$

POLES: 0, -10
ZEROS: ∞, ∞
} MUST BE EQUAL #! $\rightarrow$ STARTS @ 0, -10, ENDS @ ∞, ∞

2 POLES = 2 BRANCHES

ON REAL AXIS LEFT OF ODD # OF FINITE POLES / ZEROS ON ROOT LOCUS

$$\sigma_A = \frac{(0-10)-0}{2-0} = -5$$

11/14/13

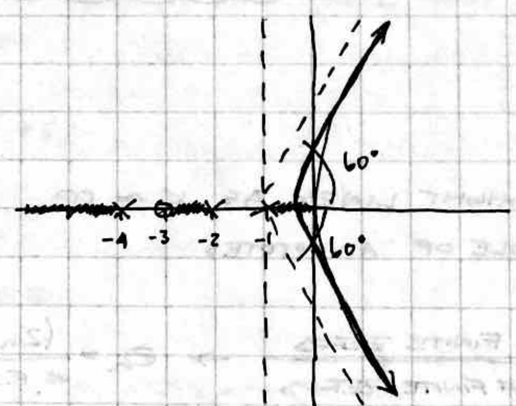
$$G(s) = \frac{(s+3)(s+4)}{(s+1)(s+2)} \rightarrow$$ POLES: $-1, -2$
ZEROS: $-3, -4$

$$G(s) = \frac{(s+3)}{s(s+1)(s+2)(s+4)} \rightarrow$$ 4 POLES: $0, -1, -2, -4$
4 ZEROS: $-3, \infty, \infty, \infty$

4 POLES $\rightarrow$ 4 BRANCHES, SYMMETRIC

3 INFINITE ZEROS $\rightarrow$ 3 ASYMPTOTES

$$\sigma_A = \frac{\Sigma \text{ POLES} - \Sigma \text{ ZEROS}}{\# F.P + \# F.Z} = \frac{-7-(-3)}{4-1} = \frac{-4}{3}$$



BREAK-AWAY / BREAK-IN PTS

WHERE DOES ROOT LOCUS LEAVE / RE-ENTER REAL-AXIS

$$\frac{\partial}{\partial s}\left[\frac{1}{G(s)H(s)}\right] = 0 \rightarrow 1 + KGH = 0, \quad KGH = -1, \quad K = \frac{-1}{GH}, \quad \frac{\partial}{\partial \sigma} = 0$$

11/14/2013

7.) jw CROSSING → WHEN DOES THE SYSTEM GO UNSTABLE?

→ ROUTH TABLE ←

1.) FORCE A ROW OF ZEROS AND GET GAIN K

2.) UP ONE ROW TO PREVIOUS POLYNOMIAL AND SET TO ZERO TO GET $\pm j\omega$

$$\text{EX. } \frac{(s+3)}{s^4+7s^3+14s^2+8s} \xrightarrow{\text{COLLAPSE FEEDBACK}} T(s) = \frac{K(s+3)}{s^4+7s^3+14s^2+8s+Ks+3K}$$

Z.)

s^4 : 1   14   3K

s^3 : 7   8+K

s^2 : $\frac{90-K}{7}$ 21K

s^1 : $\underbrace{\frac{-K^2-65K+720}{90-K}}$

↳ SET TO ZERO

$$b_2 = -\frac{\begin{vmatrix} 1 & 14 \\ 7 & 8+K \end{vmatrix}}{7} = \frac{90-K}{7}$$

$$-K^2-65K+720=0$$

$$K = -\cancel{75}, 9.65$$

NOT GREATER THAN 0

AUX. POLYNOMIAL:

$$(90-K)s^2+21K=0 \rightarrow 80s^2+210=0 \rightarrow \sqrt{s^2} = \sqrt{\frac{-210}{80}} \approx 1.59j$$

ANGLE OF DEPARTURE:

(SUM OF ANGLES FROM ZEROS) - (POLES TO ANY POINT)

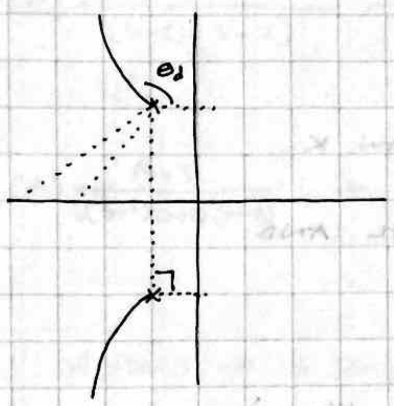
$$\sum \theta_{z_i} - \sum \theta_{p_i} = 180°$$

EX. $\frac{(s+2)}{(s+3)(s^2+2s+2)}$ 3 POLES, $-3, -1\pm j$

3 ZEROS, $-2, \infty, \infty$

$$\sigma_A = \frac{(-1+j+1-j-3)-(-2)}{3-1} = -3/2$$

11/19/2013



$\theta_3 - \theta_1 - \theta_2 - \theta_4 = 180°$

$\hookrightarrow = \theta_d$

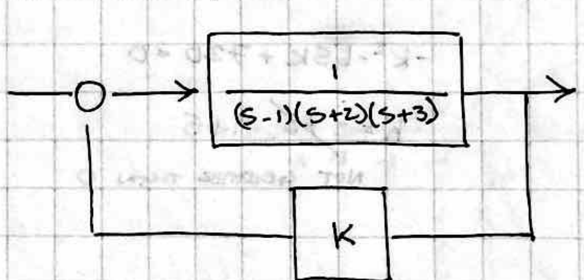
$\tan^{-1}(1) - \theta_d - 90 - \tan^{-1}(1/2) = 180$

$\theta_d = 108.4°$

11/19/2013

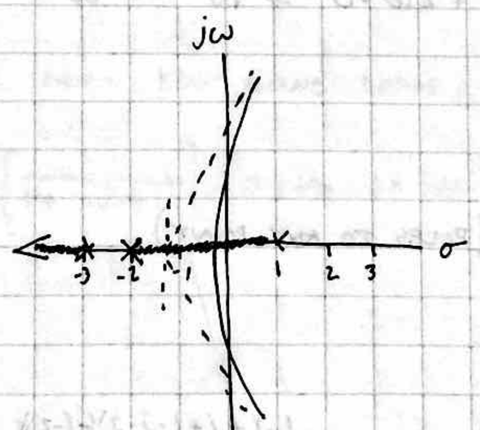
- QUIZ ON S.S. ERROR / PID CONTROL! (LONGER)

* ROOT LOCUS *



$G(s)H(s) = \frac{K}{(s-1)(s+2)(s+3)}$

1.) POLES: 1, -2, -3
2.) ZEROS: ∞, ∞, ∞
3.) BRANCHES: 3
4.) REAL AXIS (ODD # OF POLES STUFF)
5.) ASYMPTOTES / ANGLES
6.) BREAK AWAY POINTS (NEXT PAGE)



*START @ ∞ ON REAL AXIS
1^{st} POSITION IS PAST 1^{st} POLE

$$\sigma_A = \frac{\Sigma F.P - \Sigma F.Z}{\# F.P - \# F.Z}$$

$$\sigma_A = \frac{(+1-2-3)-0}{3-0} = -4/3$$

↳ DOESN'T INDICATE WHERE R.L BREAKS FROM $\mathbb{R}$

$\theta_A = \left(\frac{2\pi+1}{3}\right)\pi \rightarrow$ $n=0: \pi/3$

$n=1: \pi$

$n=2: \frac{5\pi}{3}$

6.) B-A POINTS:

$$\frac{\partial}{\partial s}\left[\frac{1}{G(s)H(s)}\right] = \frac{\partial}{\partial s}\Big[(s-1)(s+2)(s+3)\Big] = \frac{\partial}{\partial s}\Big[s^3 + 4s^2 + s - 6\Big]$$

$$= 3s^2 + 8s + 1 = 0 \rightarrow s_{1,2} = -.131 \checkmark \quad \underbrace{-2.54}_{\times}$$

$\times \rightarrow$ CANT USE, NOT ON ROOT LOCUS

$\rightarrow$ ALWAYS DEPARTS REAL AXIS @ $90°$!

7) $j\omega$ CROSSING

- ROUTH TABLE FOR C.L SYSTEM

$$\frac{G(s)}{1 + G(s)H(s)} = \frac{\frac{1}{(s-1)(s+2)(s+3)}}{1 + \frac{K}{(s-1)(s+2)(s+3)}} \rightarrow s^3 + 4s^2 + s - 6 + K$$

s^3: 1   1

s^2: 4 $K-6$ $\dfrac{-\begin{vmatrix} 1 & 1 \\ 4 & K-6 \end{vmatrix}}{4} =$

s^1: $\dfrac{10-K}{4} \rightarrow$ MULT. BY 4 $\rightarrow 10-K$

s^0: $K-6$

$$\left.\begin{matrix} K > 6 \\ K < 10 \end{matrix}\right\} \rightarrow 6 < K < 10$$

$6 < K < 10$

$\rightarrow$ FIND $j\omega$ CROSSING:

(AUXILLARY POLY.) $\rightarrow 4s^2 + K - 6 = 0$

$s = \pm j$

TAKE FROM s^2 ROW

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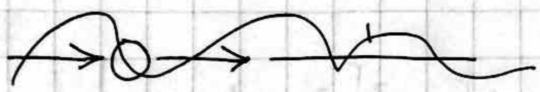
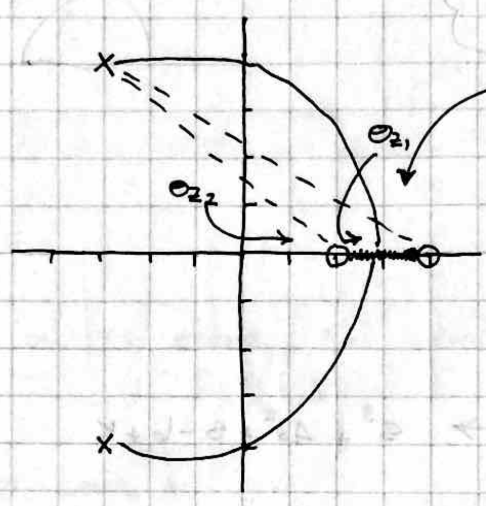


$$G(s)H(s) = \frac{K(s-2)(s-4)}{s^2+6s+25}$$

1.) POLES: $-3 \pm 4j$

ZEROS: 2, 4



ZEROS IN RHP → $j\omega$ CROSSING

C.L TF → $TF_{CL} = \dfrac{K(s-2)(s-4)}{s^2+6s+25+K(s-2)(s-4)}$

$= s^2+6s+25+K[s^2-6+8]$

$= \underbrace{(1+K)s^2 + (6-6K)s + (25+8K)}_{\text{ROUTH TABLE}}$

s^2: $1+K$ $\quad$ $25+8K$

s^1: $6-6K$ → $6-6K=0$ → $K=1$

s^0: $25+8K$

$2s^2+25+8K$ → $s = \pm 4.06j$

2.) BREAKIN POINTS

$$\frac{d}{ds}\left[\frac{s^2+6s+25}{(s-2)(s-4)}\right] = 0 \rightarrow 6s^2+17s-99=0 \rightarrow \boxed{s_{1,2} = -5.71, 2.8}$$

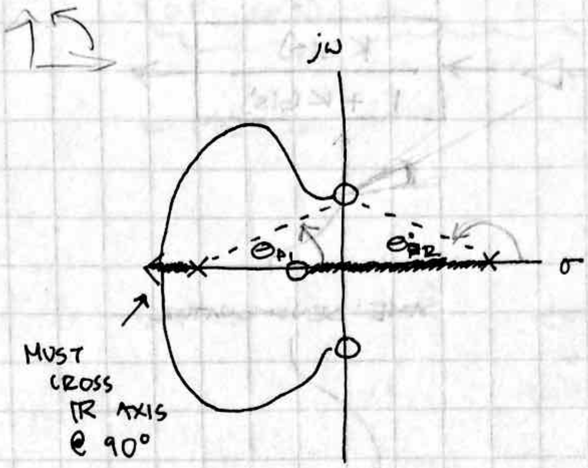
TO FURTHER DEFINE SKETCH → ANGLE OF DEPARTURE

$\theta_d = \Sigma\theta_{POLES} - \Sigma\theta_{ZEROS} - 180$

$\theta_d = 90 - \tan^{-1}[4/7] - \tan^{-1}[4/5] - 180°$

11/19/2013

$$H(s)G(s) = \frac{K(s+1)(s^2+2)}{(s-3)(s+3)} \rightarrow$$ POLES @ $3, -3, \infty$
ZEROS @ $-1, \pm\sqrt{2}\,j$



MUST CROSS ℝ AXIS @ 90°

ASYMPTOTES:

$$\sigma_a = \frac{(3-3) - (-1 + \sqrt{2}\,j - \sqrt{2}\,j)}{2-3} = -1$$

$$\theta = \frac{(2n+1)}{2-3} \quad n=0, \ -180°$$

$\theta_{P_1} = \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

$\theta_{P_2} = \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

$\theta_{z_1} = \tan^{-1}\left(\sqrt{2}\right)$

$\theta_{z_2} = 90°$

$$\boxed{\theta_d = \sum \theta_P - \sum \theta_Z + 180°}$$

$$= \theta_{P_1} + \theta_{P_2} - \theta_{Z_1} - \theta_{Z_2} + 180°$$

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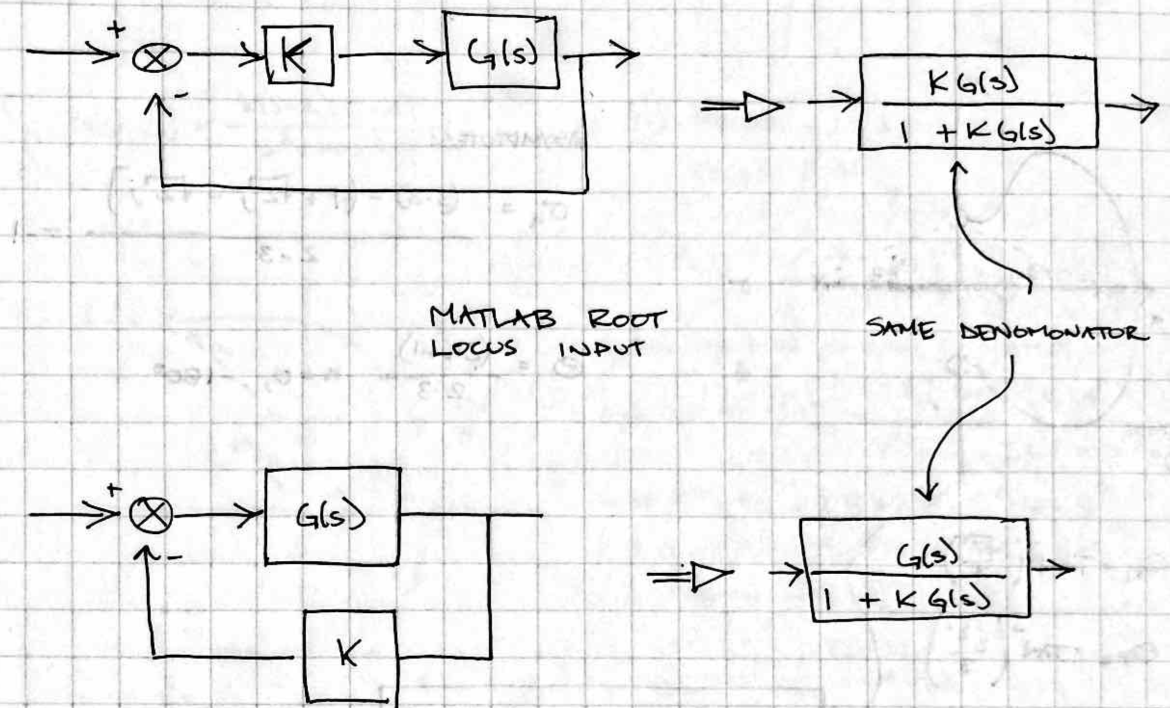
ONLINE VIDEO NOTES:

TYPICAL TEXTBOOK ROOT LOCUS PROBLEM:



IF GIVEN A TS <u>NOT</u> IN STANDARD FORM:

$$\frac{s^2+s+1}{s^3+4s^2+Ks+1} \rightarrow \frac{s^2+s+1}{\underbrace{s^3+4s^2+1+Ks}}$$

$$\rightarrow \frac{s^3+4s^2+1}{s^3+4s^2+1} + \frac{Ks}{s^3+4s^2+1} \rightarrow 1 + K\left(\frac{s}{s^3+4s^2+1}\right)$$

* HOW TO PLOT:

ALWAYS START FROM THIS FORM $\rightarrow 1 + KG(s) = 0 \rightarrow 1 + K\frac{Q(s)}{P(s)} = 0$

RULE 1.) THERE ARE n LINES (LOCI), WHERE n IS THE DEGREE OF P, Q, WHICH EVER IS GREATER

RULE 2.) AS K INCREASES FROM $0 \rightarrow \infty$, THE ROOTS MOVE FROM THE POLES OF G(s) TO THE ZEROS OF G(s)

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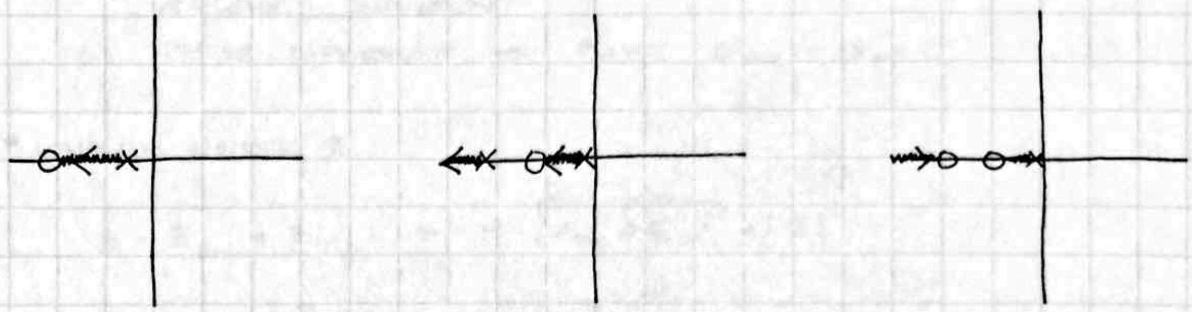
RULE 2.) CONT.

CLOSED LOOP POLES TRAVEL FROM POLES OF G(s) TO ZERO'S OF G(s)

IF P(s) = Q(s) IF P(s) > Q(s) IF P(s) < Q(s)



"IF POLES/ZEROS DON'T HAVE FRIENDS, THEY GO OFF INTO INFINITY LOOKING FOR THEM"

RULE 3.) WHEN ROOTS ARE COMPLEX, THEY OCCUR OCCUR IN CONGUGATE PAIRS

SISO TOOL

RULE 4.) AT NO TIME WILL THE SAME ROOT CROSS OVER ITS PATH

RULE 5.) THE PORTION OF $\mathbb{R}$ TO THE LEFT OF AN ODD # OF OPEN LOOP POLES AND ZEROS ARE PART OF THE LOCi
- ALWAYS COUNTED FROM RIGHT TO LEFT

RULE 6.) LINES LEAVE (BREAKOUT) & ENTER (BREAK IN) @ 90°

RULE 7.) IF THERE ARE NOT ENOUGH POLES / ZEROS TO MAKE A PAIR, THEN THE EXTRA LINES GO TO / FROM INFINITY

RULE 8.) LINES GO TO INFINITY ALONG ASYMPTOTES
- ANGLES OF ASYMPTOTES, $\theta_n = \frac{(2q+1)}{n-m} \cdot 180°$ $q = 0, 1, 2$
- CENTROID OF ASYMPTOTES, $\sigma_n = \frac{\Sigma FP - \Sigma FZ}{n-m}$

RULE 9.) IF THERE IS AT LEAST TWO LINES TO INFINITY, THEN THE SUM OF ALL ROOTS IS CONSTANT

RULE 10.) K GOING FROM 0 TO NEG. INFINITY CAN BE DONE BY REVERSING RULE 5 & ADDING 180° TO THE ASYMPTOTE ANGLES

11|21|2013

BODE PLOTS:

$$\tan^{-1}\left\{\frac{2\zeta}{-2\zeta^2+\sqrt{(1+4\zeta^4)}}\right\}$$

- FREQUENCY RESPONSE
  - FOR A LINEAR SYSTEM
    - SINUSOIDAL INPUT → SINUSOIDAL OUTPUT
    - FREQ. SAME
    - AMPLITUDE: DIFFERENT
    - PHASE DIFFERENT → PHASE $\phi_{out} - \phi_{in}$

- COMPLEX NUMBER Z

$$Z = Z_{Re} + Z_{Im} j \;\rightarrow\; M = \sqrt{Z_{RE}^2 + Z_{IM}^2} = |Z|$$

$$\phi = \tan^{-1}\left(\frac{Z_{IM}}{Z_{RE}}\right)$$

TO PLOT:

(1.) REPLACE S W/ $j\omega$

(2.) FIND REAL & IMAG. PARTS (MAKE DENOM REAL)

(3.) FIND MAGNITUDE & ANGLE

REPRESENTATION: $Z = |Z|\angle\phi$

EX.

$$G(s) = \frac{1}{s+2} \rightarrow G(j\omega) = \frac{1}{j\omega+2} \rightarrow \text{(REAL \& IMAG PARTS)}$$

MULT. BY $2-j\omega$ $\rightarrow$ $G(j\omega) = \frac{2-j\omega}{(2-j\omega)(2+j\omega)} = \frac{2-j\omega}{4+\omega^2} = \frac{2}{4+\omega^2} + \frac{j\omega}{4+\omega^2}$

$$M = \left\{\left(\frac{2}{4+\omega^2}\right)^2 + \left(\frac{\omega}{4+\omega^2}\right)^2\right\}^{1/2} = \left\{\frac{4+\omega^2}{(4+\omega^2)^2}\right\}^{1/2} = \frac{1}{\sqrt{4+\omega^2}}$$

$$\phi = \tan^{-1}\left[\frac{-\omega/4+\omega^2}{2/4+\omega^2}\right] = \tan^{-1}\left(\frac{-\omega}{2}\right)$$

11|21|2013

$20\log_{10} M \rightarrow dB$

$20\log(2x) = 20\log 2 + 20\log x = 6 + 20\log x$

$20\log(10x) = 20\log 10 + 20\log x = 20\log x + 20$

→ TO DRAW A BODE PLOT ←

- log MAGNITUDE & PHASE SHIFT PLOTS
- STRAIGHT LINE APPROX

$$G(j\omega) = \frac{k(j\omega + z_1)(j\omega + z_2)}{(j\omega + p_1)(j\omega + p_2)}$$

$$20\log|G(j\omega)| = 20\log k + 20\log(j\omega + z_1) + 20\log(j\omega + z_2)$$
$$-20\log(j\omega + p_1) - 20\log(j\omega + p_2)$$

RESPONSE TO $G(s) = \frac{s+a}{a} \rightarrow G(j\omega) = \frac{j\omega + a}{a} = 1 + \frac{j\omega}{a}$

FOR SMALL ω: $G(j\omega) \approx 1$

$20\log|G| = 20\log(1) = 0$

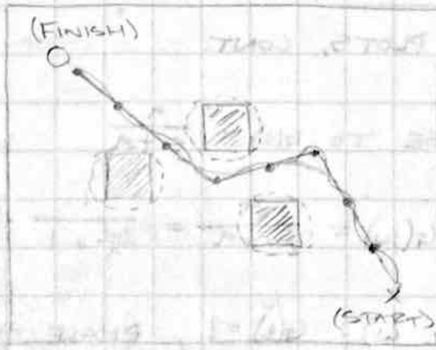
FOR LARGE ω: $\omega \gg a$

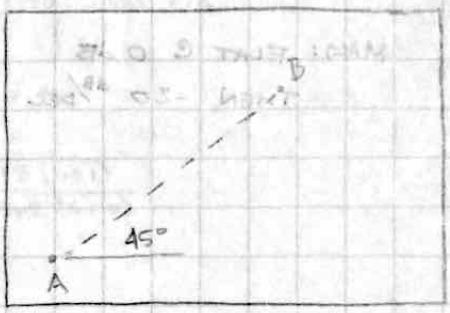
$$G(j\omega) \approx \frac{j\omega}{a} = 20\log\left(\frac{\omega}{a}\right)$$

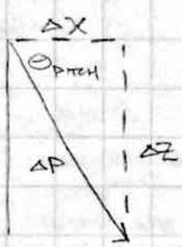
IF $\omega < a$, USE SMALL ω } a IS THE
IF $\omega > a$, USE LARGE ω }

QUESTIONS FOR DR. HOLLINGER

- SIMILARITY OF SIMULINK MODELS
- DIFFERENT PID TUNING METHODS
- IS THE HEADING CONTROL SYS. A PID?
- INCLUDE DISTURBANCES?
- GO FROM CONTROL SYSTEM TRANSIENT RESPONSE TO REAL WORLD ERROR
- MODELING DISTURBANCES (IE. WATER CURRENT)
- WHERE DOES THE RUDDER ACTUATOR TERM COME FROM







COUPLE HEADING & PITCH

PLOT THROUGH DIFFERENT VALUES OF GAIN

OPTIMIZE EACH GAIN, HOLD CONSTANT, RUN THROUGH EACH OTHER GAIN

*GENERAL NOTES: *

-

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* EXTRA CREDIT * (THURSDAY OF NEXT WEEK) +3 POINTS APPLIED TO FINAL
- EMAIL PROBLEM (OR A COUPLE OF PROBLEMS TO HOLLINGER

* BODE PLOTS, CONT.

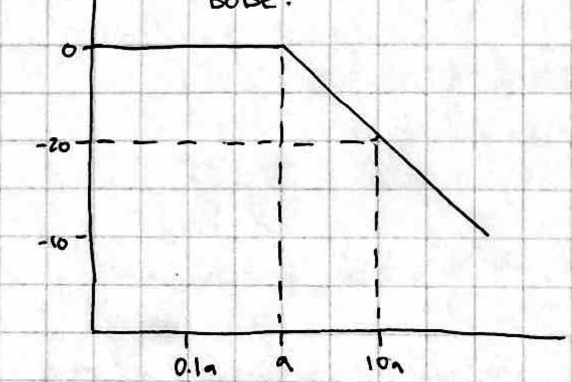
RESPONSE TO $G(s) = \frac{a}{s+a}$

$$\rightarrow G(j\omega) = \frac{a}{j\omega + a} = \frac{1}{\frac{j\omega}{a}+1} \cdot K = \frac{1}{\left(\frac{j\omega}{a}+1\right)} \cdot \frac{a}{a}$$

(SMALL ω): $G(\overset{j\omega}{\cancel{s}}) = 1$, PHASE = $0°$, MAGNITUDE = $20 \log = 0$

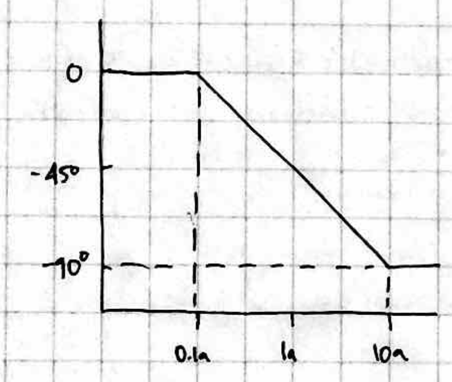
(LARGE ω): $G(j\omega) = \frac{1}{j\omega/a} = -j\frac{a}{\omega}$, PHASE = $90°$

BODE:



MAG: FLAT @ 0 dB
THEN -20 dB/DEC

PHASE:



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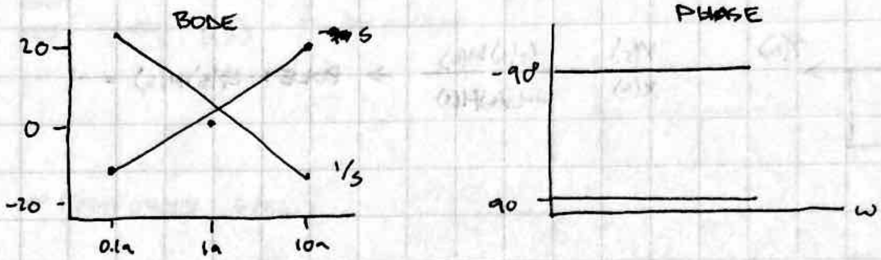
RESPONSE TO $G(s) = s$

$$G(j\omega) = j\omega$$

MAGNITUDE $= 20\log(j\omega) = 20\log\omega$, PHASE $= 90°$



RESPONSE TO $G(s) = 1/s$

PHASE = 90

$$G(s) = \frac{15(s+1)}{(s+2)(s+10)}$$

UNITY FB SYS.

$G(s)$ = OPEN LOOP TRANS. FUNCTION

$$T(s) = \frac{G(s)}{1+G(s)}$$ (CLOSED LOOP TRANS. FUNCTION)

DERIVATION:

$$T(s) = \frac{KG(s)}{\underbrace{1+KG(s)H(s)}_{\text{WHAT WE CARE ABOUT}}}$$

$\rightarrow$ let $G(s) = \frac{(s+2)}{(s+1)}$, $H(s) = 1$ $\rightarrow$ $1 + K\left(\frac{s+2}{s+1}\right) = 0$

$(s+1) + K(s+2) = 0 \rightarrow s = -1$

$\rightarrow$ MORE BODE PLOTS $\leftarrow$

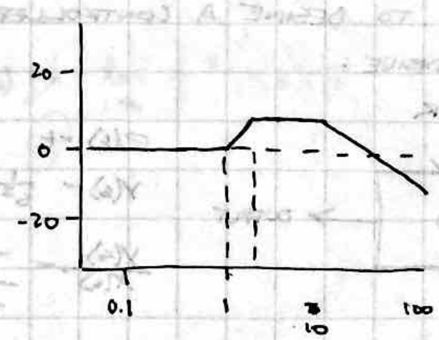
$$G(s) = \frac{15(s+1)}{(s+2)(s+10)} \rightarrow G(j\omega) = \frac{15}{2\cdot 10}\left(\frac{1+j\omega}{(\frac{j\omega}{2}+1)(\frac{j\omega}{10}+1)}\right) \rightarrow K = 3/4$$

BREAKPOINTS:

ZERO @ $s = -1$ } 1

POLE @ $s = -2$ } 2

POLE @ $s = -10$ } 10



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STABILITY (AGAIN):

STABLE IF ALL ROOTS IN DENOM. ARE IN LHP.

→ HOW STABLE IS SYSTEM

$$\frac{Y(s)}{X(s)} = \frac{G(s)H(s)}{1+G(s)H(s)} \rightarrow \text{POLE: } G(s)H(s) = -1$$

$$G(j\omega)H(j\omega) = -1$$

→ STABILITY ←

$$G(s) = \frac{K}{(s+5)(s+20)(s+50)} \rightarrow G(j\omega) = \frac{K}{(j\omega+5)(j\omega+20)(j\omega+50)} =$$

$$\frac{K}{(5)(20)(50)} \cdot \frac{1}{\left(\frac{j\omega}{5}+1\right)\left(\frac{j\omega}{20}+1\right)\left(\frac{j\omega}{50}+1\right)}$$

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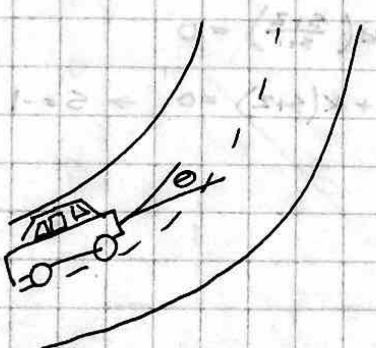
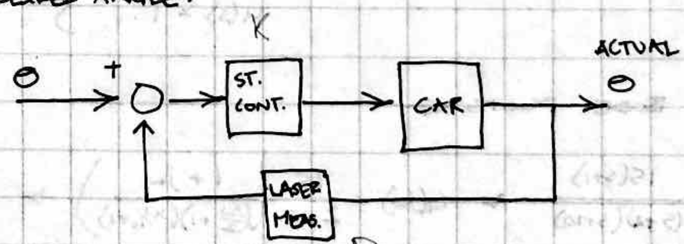
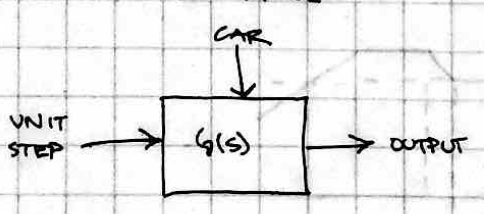
END TO END EXAMPLE:



DESIRED ANGLE:



→ NEED TO DESIGN A CONTROLLER: TAKE FOR A TEST DRIVE:



$\theta(t) = t$

$Y(s) = \frac{1}{s^2}$

$$\frac{Y(s)}{X(s)} = \frac{\frac{1}{s^2}}{\frac{1}{s}} = \frac{1}{s}$$

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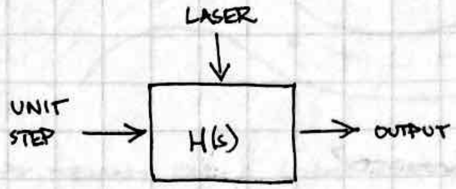
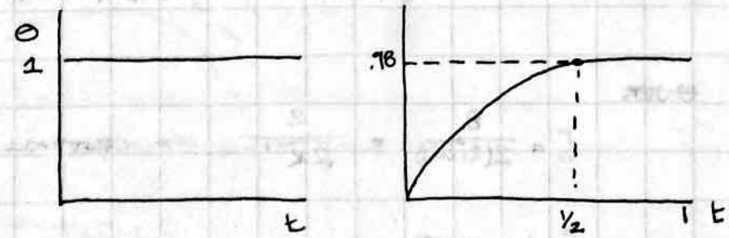
2.) SYSTEM ID ON LASER MEASUREMENT



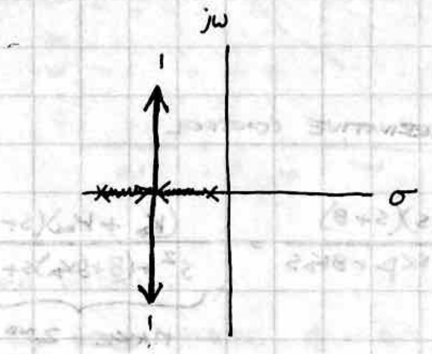


→ FIRST ORDER SYSTEM:

$$H(s) = \frac{a}{s+a} = \frac{8}{s+8} \rightarrow T_{s_{2\%}} = \frac{4}{a} = \frac{1}{2} \quad (a=8)$$

→ TRY PROPORTIONAL CONTROL SYSTEM

ROOT LOCUS: $G(s)H(s) = \frac{1}{s}\left(\frac{8}{s+8}\right) = \frac{8}{s(s+8)} \rightarrow$ POLES @ $0, -8$; ZEROS @ ∞, ∞



$$\sigma_A = \frac{\Sigma FP - \Sigma FZ}{\#FP - \#FZ} = \frac{-8}{2} = -4$$

$$\theta_A = \frac{180(2n+1)}{2} \rightarrow \theta = 90°, 270°$$

→ FIND T(s):

$$T(s) = \frac{KG(s)}{1+G(s)H(s)} = \frac{K/s}{1+K/s\left(\frac{8}{s+8}\right)} = \frac{K(s+8)}{s^2+8s+8K}$$

→ S.S. ERROR

$$e_{ss} = 1 - \lim_{s\to 0} sY(s) = 1 - \lim_{s\to 0} \frac{s}{s} K \frac{(s+8)}{s^2+8s+8K} = 1-1=0$$

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→ SECOND ORDER APPROX.

$$T(s) \approx \frac{8K}{s^2+8s+8K} \rightarrow \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

~~OVER~~

$$\zeta = \frac{8}{2(2\sqrt{2K})} = \frac{2}{\sqrt{2K}} \rightarrow \text{CRITICALLY DAMPED?}$$

$$\zeta = 1 \;,\; K = 2$$

→ BEHAVIOR:

$(\zeta > 1)$ FOR $K < 2$ (OVERDAMPED)

$(0 < \zeta < 1)$ FOR $K > 2$ (UNDERDAMPED)

→ WHAT K GIVES 16% OS

* BUT WE WANT $T_{s_{2\%}} < 1$ ☹ → ADD DERIVATIVE CONTROL

$$\frac{Y(s)}{X(s)} = T(s) = \frac{(K_P + K_D s)\cdot \frac{1}{s}}{1 + (K_P + K_D s)\cdot(\frac{1}{s})\cdot(\frac{8}{s+8})} = \frac{(K_P + K_D s)(s+8)}{s^2 + 8s + 8K_P + 8K_D s} = \underbrace{\frac{(K_P + K_D)(s+8)}{s^2 + (8+8K_D)s + 8K_P}}_{\text{MAKE 2ND ORDER APPROX.}}$$

$$\omega_n^2 = 8K_P \rightarrow T_{s_{2\%}} = \frac{4.0}{\zeta\omega_n} < 1$$

$$\zeta = \frac{4+4K_D}{2\sqrt{2K_P}} = \frac{2+2K_D}{\sqrt{2K_P}}$$

I WANT 16% O.S.

$$T_{s_{2\%}} = 0.5\text{ SEC} \rightarrow T_{s_{2\%}} = \frac{4}{\zeta\omega_n}$$

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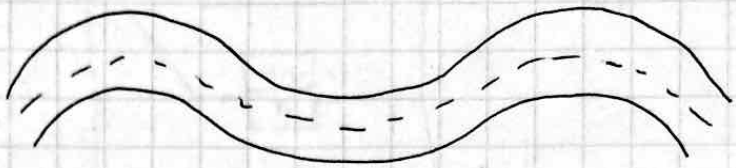
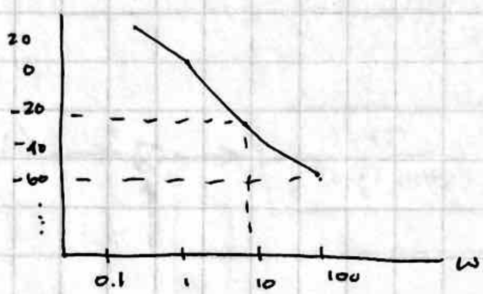


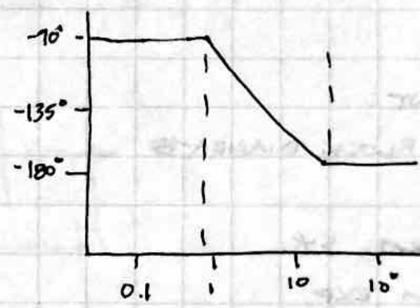
→ DRIVING ON A CURVY ROAD? → NEED A SINUSOIDAL INPUT

$$G(j\omega)H(j\omega) = \frac{8}{j\omega(j\omega+8)} = \frac{8}{8}\underbrace{\left(\frac{1}{j\omega\left(\frac{j\omega}{8}+1\right)}\right)}_{\to \text{ STD. FORM}}$$

2 POLES:

$\frac{1}{j\omega} \to$ CONST. -20 dB/DEC $\qquad \frac{j\omega+1}{8} \to -20$ dB/DEC @ $\omega = 8$





GAIN MARGIN:

→ FIND WHERE $\phi = 180°$, BUT PLOT IS ASYMPTOTIC TO $180°$...

GAIN MARGIN = INF!

PHASE MARGIN:

→ FIND 0 dB → $-180° - 95° - \underline{85°}(-1)$ → NEG. DOESN'T MATTER

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* FINAL EXAM *

4 QUESTIONS

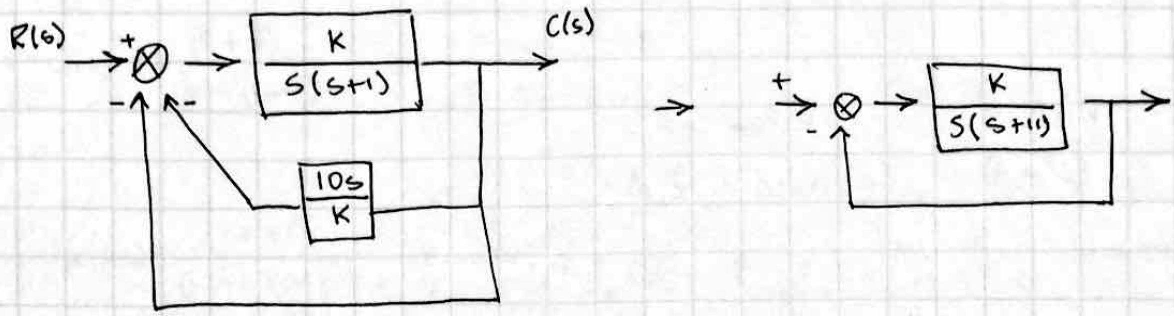
1.) PID CONTROL (SIMILAR TO LAST QUIZ)
- KNOW 2ND ORDER SYS, S.S. ERROR, FEEDBACK

2.) ROOT LOCUS PROBLEM
- BREAKAWAY PTS, ASYMP., θ ARRIVE / DEPART

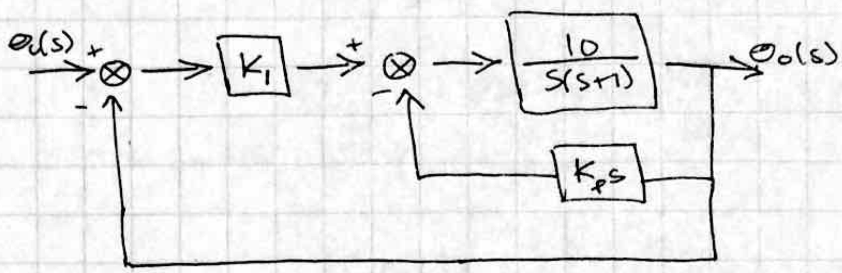
3.) BODE PLOTS (SKETCH, PHASE / GAIN MARGIN)

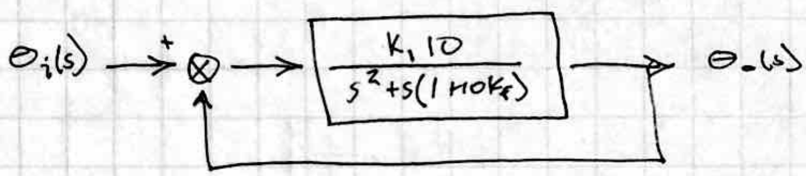
4.) INTEGRATED FUN!
- STABILITY, 2ND ORDER SYS, FEEDBACK, ROOTLOCUS, F.V.T (LARGELY CONCEPTUAL)

NOT AS IMPORTANT:
- NO GNARLY BLOCK DIAGRAMS
- STATE SPACE
- MECH / ELECTRICAL SYS.
- PARTIAL FRAC EXP.



CH.7 #35





$$K_v = \lim_{s\to 0} sG(s) = \frac{K_1 10}{1+10K_f} = 10$$

$$\frac{10K_1}{s^2+s(1+10K_f)+10K_1} = T(s) \rightarrow \omega_n = \sqrt{10K_1}$$

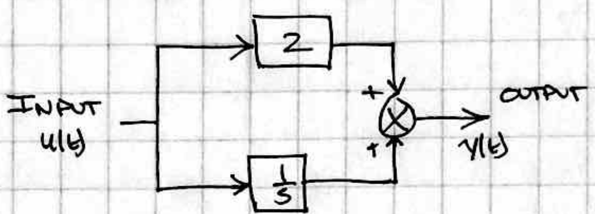
BODE PLOT NOTES:

$1\,TU = 10\log_{10} \Delta POWER \rightarrow 1\,TU = 1$ DECIBEL

POWER $\propto$ AMPLITUDE^2, SO 1 DECIBEL $= 10\log_{10} AMP^2$

$\rightarrow$ PROP. OF LOGS: $2 \cdot 10 \log_{10}(AMP) \rightarrow \boxed{20\log_{10}(AMP)}$

$\hookrightarrow$ FOR CONVERTING AMPLITUDE TO DECIBELS



$$y(t) = 2u(t) + \frac{u(t)}{s}$$

$$y(t) = (2 + 1/s)\,u(t)$$

$$\boxed{\frac{y(t)}{u(t)} = \frac{2s+1}{s}}$$

$$s = \underbrace{\cancel{\sigma}}_{=0 \text{ FOR TRANSIENT RESPONSE}} + j\omega$$

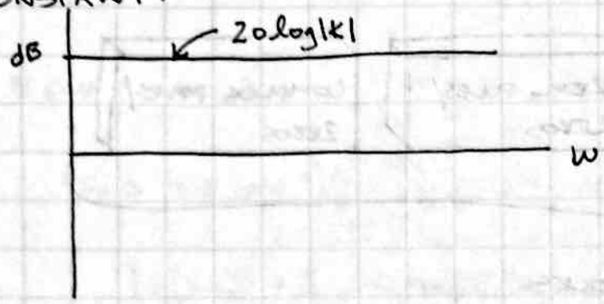
SUBSTITUTE: $TF(j\omega) = \dfrac{2j\omega + 1}{j\omega} = 2 + \frac{1}{\omega} j$

$M = \{REAL^2 + IMAG^2\}^{1/2}$

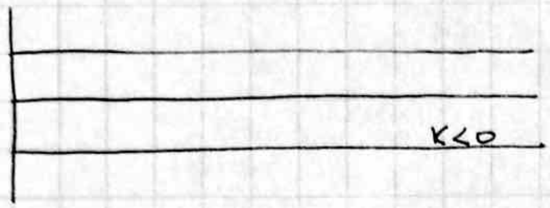
$\phi = ATAN(IMAG/REAL)$

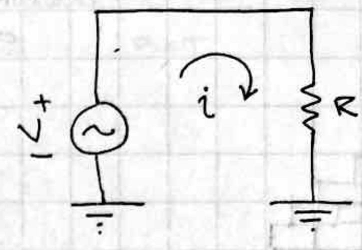
BODE PLOTS CONT.

CONSTANT!



GAIN IS CONSTANT FOR ALL FREQUENCIES.





$V = iR \rightarrow V(t) = i(t)R$

$V(s) = i(s)R \rightarrow \boxed{\frac{i(s)}{V(s)} = \frac{1}{R}}$

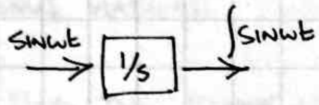
→ TRANSFER FUNCT.

CONST. GAIN = $1/R$

W 0° ϕ SHIFT

* POLES AND ZEROS @ ORIGIN *

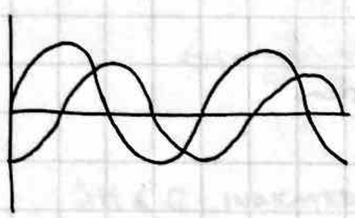
EXAMPLE:

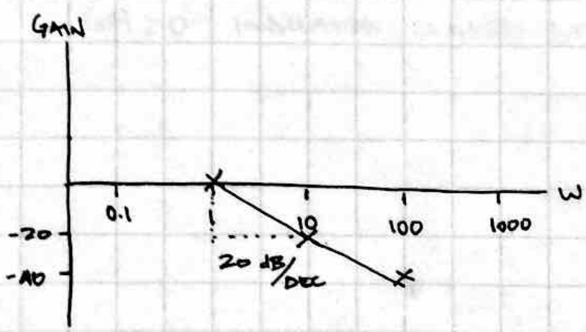


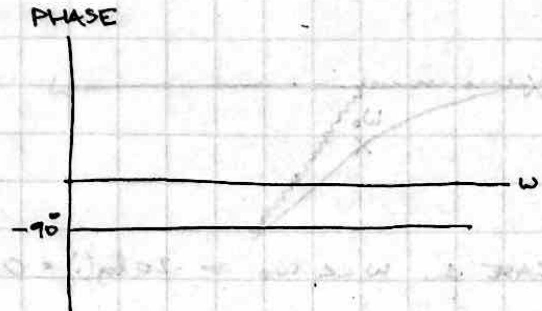
$\int \sin\omega t = -\left(\frac{1}{\omega}\right)\cos\omega t$

PHASE

GAIN





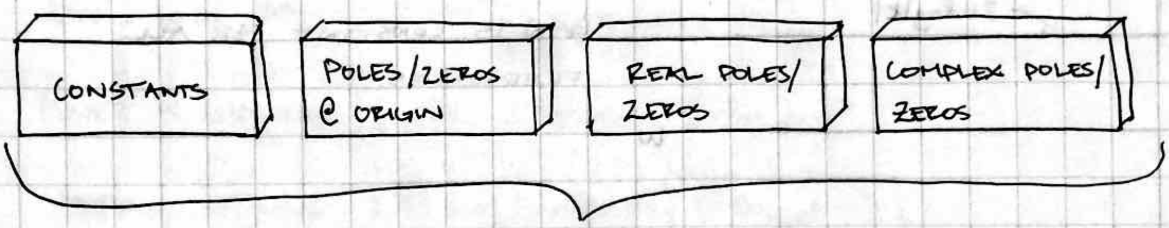


$w = 1 \rightarrow GAIN = \frac{1}{1} = 1 \rightarrow 20\log 1 = 0$

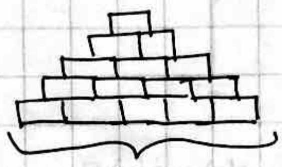
$w = 10 \rightarrow GAIN = \frac{1}{10} = 0.1 \rightarrow 20\log(0.1) = -20$

$w = 100 \rightarrow GAIN = \frac{1}{100} = 0.01 \rightarrow 20\log(0.01) = -40$

BODE PLOTS CONTS:



BUILDING BLOCKS
OF BODE
PLOTS!



THIS IS A TRANSFER FUNCTION → FREQ. RESPONSE CAN BE OBTAINED BY SUMMING THE RESPONSE OF INDIVIDUAL COMPONENTS

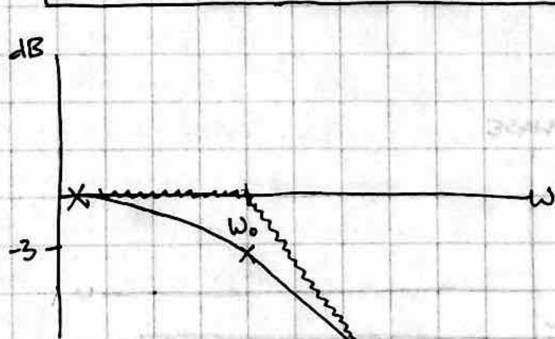
* TRANSFER FUNCTION W/ A SINGLE REAL POLE *

$$\boxed{= \frac{1}{1 + \frac{s}{\omega_0}}}$$

ω_0 IS THE BREAK OR "CORNER FREQ."

$$\boxed{-20\log_{10}\left(\sqrt{1 + \frac{\omega^2}{\omega_0^2}}\right)} \leftarrow \text{GAIN EQUATION}$$

$$\boxed{\phi = \text{ATAN}\left(\frac{\text{IMAG}}{\text{REAL}}\right) = \text{ATAN}\left(\frac{-\omega}{\omega_0}\right)} \leftarrow \text{PHASE EQUATION}$$



CASE 1: $\omega \ll \omega_0 \rightarrow -20\log(1) = 0$ dB

CASE 2: $\omega = \omega_0 \rightarrow -20\log(\sqrt{2}) = -3$ dB

CASE 3: $\omega \gg \omega_0 \rightarrow -20\log_{10}\left(\frac{\omega}{\omega_0}\right) = -20$ dB/DEC

GAIN AND PHASE MARGINS

* PHASE MARGIN:

- FIND THE GAIN CROSSOVER FREQUENCY W_g → SOLVED BY : $|G(jW_g)| = 1$

$|G(jW_g)| = 1$ HOWEVER IS 0dB ON THE BODE PLOT, SO SIMPLY FIND W_g WHERE $dB = 0$ ON MAG. PLOT.

* GAIN MARGIN:

- FIND THE PHASE CROSSOVER FREQUENCY W_P → SOLVED BY $ARG(G(jW)) = -180$ SIMPLY FOUND ON BODE:

$$GM = \frac{1}{G(jW_P)} \rightarrow -20\log_{10}|G(jW_P)|$$

$GM \leq 0$ INDICATED CLOSED LOOP INSTABILITY

$GM > 0$ INDICATES CLOSED LOOP STABILITY